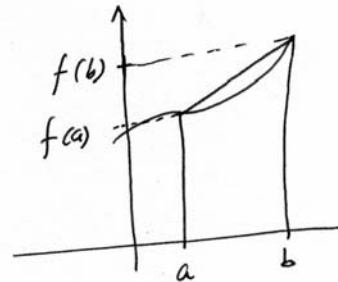


(28)

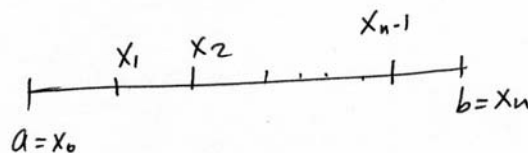
Numerical Solution of Fredholm Int. Eqs.

Review Trapezoidal rule:

$$\int_a^b f(x) dx \approx \frac{b-a}{2} \left[\frac{f(b)+f(a)}{2} \right]$$



If



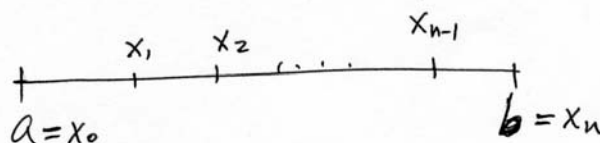
$x_i - x_{i-1} = h$
uniform partition.

$$\begin{aligned} \text{then, } \int_a^b dx &= \int_{a=x_0}^{x_1} dx + \int_{x_1}^{x_2} dx + \dots + \int_{x_{n-1}}^{x_n} dx \approx \\ &\approx \frac{h}{2} [f(a) + 2f(x_1) + \dots + 2f(x_{n-1}) + f(b)] \\ &= \frac{h}{2} \left[f(a) + \sum_{j=1}^{n-1} f(x_j) + f(b) \right]. \end{aligned}$$

Now, Consider $Ku - \lambda u = f$

$$\text{or } \boxed{\int_a^b K(x, y) u(y) dy - \lambda u(x) = f(x).} \quad (28.1)$$

Make a uniform partition of interval $[a, b]$.



Evaluate the integral equation at these points. $x = x_i$
 $i = 0, 1, 2, \dots, n$

$$\int_a^b K(x_i, y) u(y) dy - \lambda u(x_i) = f(x_i), \quad i = 0, 1, 2, \dots, n$$

Use notation $f(x_i) \rightarrow f_i$
 $u(x_i) \rightarrow u_i$

$$\int_a^b K(x_i, y) u(y) dy - \lambda u_i = f_i, \quad i = 0, 1, 2, \dots, n$$

$n+1$ eqns.

Using trapezoidal for the integral

$$h = x_i - x_{i-1}$$

$$h = \frac{b-a}{n}$$

$$\int_a^b K(x_i, y) u(y) dy \approx \frac{h}{2} [K(x_i, a) u(a) + \sum_{j=1}^{n-1} K(x_i, x_j) u_j + K(x_i, b) u(b)]$$

$$\approx \frac{h}{2} [K(x_i, a) u(a) + \sum_{j=1}^{n-1} K(x_i, x_j) u_j + K(x_i, b) u(b)]$$

Therefore, the integral eqn. (28.1) reduces to an algebraic linear system of equations: of $(n+1)$ eqns. with $(n+1)$ unknowns.

$$\frac{h}{2} [K(x_i, a) u(a) + \sum_{j=1}^{n-1} K(x_i, x_j) u_j + K(x_i, b) u(b)] - \lambda u_i = f_i$$

$i = 0, 1, 2, \dots, n$

Example: Use the trapezoidal rule to reduce

Equ. $\int_0^1 e^{x+y} u(y) dy - \lambda u(x) = f(x)$

to an algebraic system of (4) eqns. with (4) unknowns.

$$\begin{array}{c} | \quad | \quad | \quad | \\ 0=x_0 \quad \frac{1}{3}=x_1 \quad \frac{2}{3}=x_2 \quad 1=x_3 \end{array} \quad \begin{array}{l} h = \frac{1}{3} \\ x_0 = 0 \end{array}$$

$$i=0: \quad \frac{1}{6} [e^{0+0} u_0 + e^{0+1/3} u_1 + e^{0+2/3} u_2 + e^{0+1} u_3] - \lambda u_0 = f_0$$

$$i=1: \quad \frac{1}{6} [e^{1/3+0} u_0 + e^{1/3+1/3} u_1 + e^{1/3+2/3} u_2 + e^{1/3+1} u_3] - \lambda u_1 = f_1$$

$$i=3: \quad \frac{1}{6} [e^{1+0} u_0 + e^{1+1/3} u_1 + e^{1+2/3} u_2 + e^{1+1} u_3] - \lambda u_3 = f_3$$

After solving this linear system of 4 eqns. 4 unknowns,
we will have approximations for $u(x)$ at the nodes.

$$x_0, x_1, x_2, x_3$$

$$u(x_0) = u(a) \approx u_0$$

$$u(x_1) \approx u_1$$

$$u(x_2) \approx u_2$$

$$u(x_3) \approx u_3.$$