Definition of Continuous function

For functions $f: D \subset \mathbb{R} \to \mathbb{R}$

real-valued functions of 1 variable.

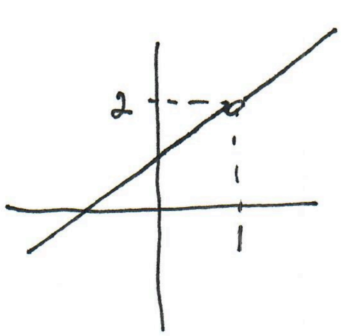
> The function f is continuous at "a" if
>
> $$\lim_{x \to a} f(x) = f(a)$$

This definition requires:

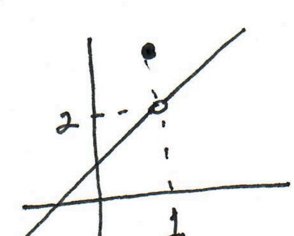
1) limit of $f(x)$ exists at $x = a$. or $\lim_{x \to a} f(x) = \ell$
2) f is defined at $x = a$
3) $\lim_{x \to a} f(x) = \ell = f(a)$.

Analize these three cases, $\lim_{x \to 1} f(x)$ ~~of~~ :

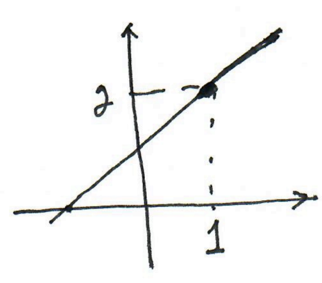
i) $f(x) = \dfrac{x^2 - 1}{x - 1}$,



ii) $f(x) = \begin{cases} \dfrac{x^2-1}{x-1}, & x \neq 1 \\ 3, & x = 1 \end{cases}$



iii) $f(x) = \begin{cases} \dfrac{x^2-1}{x-1}, & x \neq 1 \\ 2, & x = 1 \end{cases}$



38) At what points is

$$f(x,y) = \begin{cases} \dfrac{xy}{x^2+xy+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

turn for answer

Continuous..?

Use this problem to ~~discuss~~ discuss concept of Continuity.

along lines $y = mx$

#(4) a) $f(x,y) = \dfrac{x^4 - y^4}{x^2+y^2}$,

$$\lim_{x \to 0} \frac{x^4 - m^4x^4}{x^2+m^2x^2} = \lim_{x\to 0} \frac{(1-m^4)\,x^{\cancel{4}\,2}}{\cancel{x^2}(1+m^2)} = \lim_{x\to 0} \frac{x^2(1-m^4)}{1+m^2} = \frac{0}{1+m^2} = 0$$

$$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)\to(0,0)} \frac{(x^2-y^2)\cancel{(x^2+y^2)}}{\cancel{x^2+y^2}} = 0$$

Is $f(x,y)$ conts? $\quad z = x^2 - y^2, \ (x,y) \neq (0,0)$

Focus on this

b) How about cont. for

$$f(x,y) = \begin{cases} \dfrac{x^4-y^4}{x^2+y^2}, & (x,y) \neq (0,0) \\ 1, & (x,y) = (0,0). \end{cases} \quad ?$$

c) How about cont for

$$f(x,y) = \begin{cases} \dfrac{x^4-y^4}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

Good examples for Limits and Continuity

1) $$\lim_{(x,y)\to(0,0)} \frac{x^4 - y^4}{x^2 + y^2} =$$

2) $$\lim_{(x,y)\to(1,0)} \frac{xy - y}{(x-1)^2 + y^2}$$

Try $\underset{0}{x=0}$, $\underset{0}{y=0}$, and $\underset{1/2}{y=x-1}$

3) $$\lim_{(x,y)\to(2,3)} \frac{yx+y^2}{x^2+y^2}$$

4) $$\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2}$$