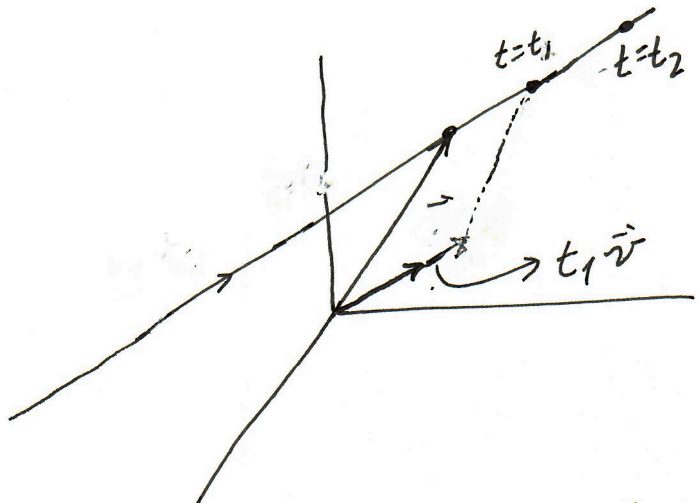
13.1 Curves in Space

Consider a line given in vector form:

$L: \quad \langle x, y, z \rangle = t \langle v_1, v_2, v_3 \rangle + \langle x_0, y_0, z_0 \rangle$

or Simply (1.1)

$\vec{r}(t) = t\vec{v} + \vec{r}_0$



Where $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$, $\vec{r}_0 = \langle x_0, y_0, z_0 \rangle$.

For the line L an appropriate function representation is given by

$$\vec{r}: \mathbb{R} \longrightarrow \mathbb{R}^3 \qquad (1.2)$$

$$t \longrightarrow \vec{r}(t) = t\vec{v} + \vec{r}_0.$$

$$= \langle tv_1 + x_0, tv_2 + y_0, tv_3 + z_0 \rangle$$

The formula (1.2) is a particular case of a "vector function" in the space (3D).

$x(t) = tv_1 + x_0$
$y(t) = tv_2 + y_0$
$z(t) = tv_3 + z_0$

In general, we will define a vector function $\left(\begin{smallmatrix}\text{in}\\ 3D\end{smallmatrix}\right)$ as

$$\vec{r}: I = [a,b] \longrightarrow \mathbb{R}^3$$
$$t \longrightarrow \vec{r}(t) = \langle x(t), y(t), z(t) \rangle$$

Domain $\longrightarrow$ Range.

Where

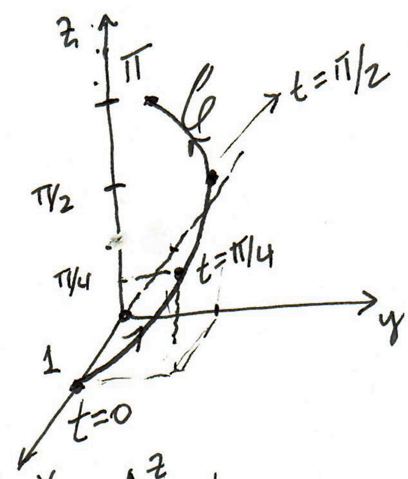
$$x: I \longrightarrow \mathbb{R}$$
$$t \rightarrow x(t)$$

$$y: I \longrightarrow \mathbb{R}$$
$$t \rightarrow y(t)$$

$$z: I \longrightarrow \mathbb{R}$$
$$t \rightarrow y(t).$$

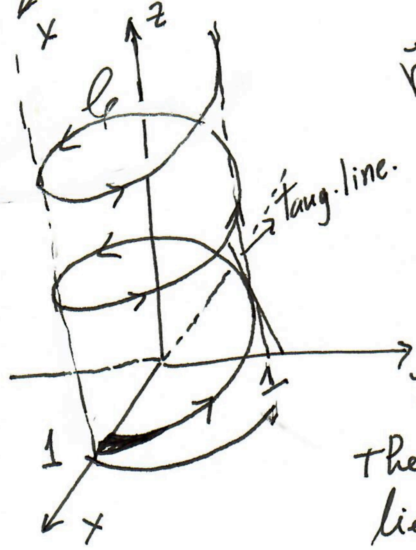
Example 2. Consider the vector function:
(Example 4 (book))

$$\vec{r}(t) = \langle \cos t, \sin t, t \rangle, \quad t \geq 0$$



t	$(x(t), y(t), z(t))$
0	$(1, 0, 0)$
$\pi/2$	$(0, 1, \pi/2)$
$\pi/4$	$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, \pi/4\right)$
π	$(-1, 0, \pi)$

Another scale:



$\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$
$\langle \cos t, \sin t, t \rangle$

Notice
$x^2 + y^2 = \cos^2 t + \sin^2 t = 1$
for all t.

Then, all points in the curve C lie on the circular cylinder $\boxed{x^2 + y^2 = 1}$

Def. (space curves)

the set C of points (x, y, z) in space that satisfy

$$x = f(t), \quad y = g(t), \quad z = h(t), \qquad t \text{ in } I = [a, b].$$

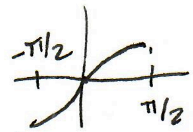
is called a space curve, if $f(t)$, $g(t)$, and $h(t)$ are continuous functions in $I = [a, b]$.

13.1 #35 (9th Ed)

#26(old) $x(t) = t\cos t$

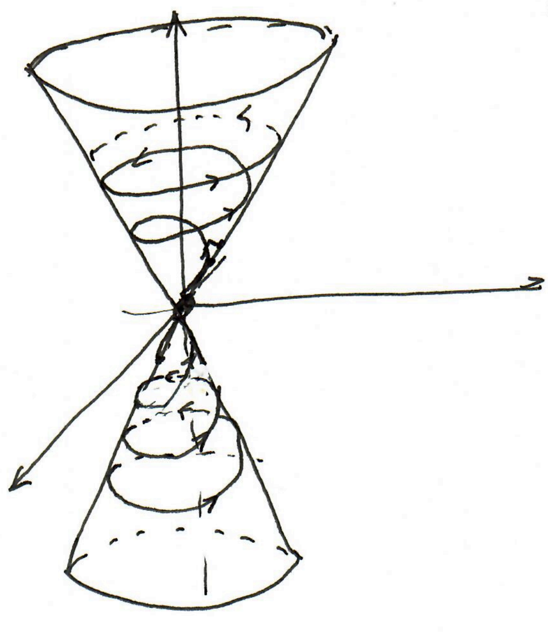
$y(t) = t\sin t$ a) Draw the curve

$z(t) = t$, $t \in \mathbb{R}$. b) Find the tangent line at $P_0\,(0, \pi/2, \pi/2)$



$$x^2 + y^2 = t^2(\cos^2 t + \sin^2 t) = t^2 = z^2$$

So, any point (x, y, z) in the curve satisfies the equ. for a cone. It means the graph of the curve lies in the cone.



When $t = 0 \Rightarrow z = 0$

$x^2 + y^2 = 0$

$\Rightarrow x = 0,\ y = 0.$

$t = \frac{\pi}{2}$

$\vec{r}(\pi/2) = \langle 0, \pi/2, \pi/2 \rangle$

$t = -\pi/2 \Rightarrow z = -\pi/2$

$\vec{r}(t) = \langle 0, -\pi/2, -\pi/2 \rangle$

$$\vec{r}\,'(t) = \langle \cos t - t\sin t,\ \sin t + t\cos t,\ 1 \rangle$$

$$r'(\pi/2) = \langle 0 - \pi/2,\ 1,\ 1 \rangle$$

Tang. line: $\vec{r}(t) = t\left\langle -\frac{\pi}{2}, 1, 1 \right\rangle + \langle 0, \pi/2, \pi/2 \rangle.$

Find two surfaces that intersect at the curve

$$\vec{r}(t) = \langle 1+t,\ 2+5t,\ -1+6t \rangle$$

$x = 1+t, \quad y = 2+5t, \quad z = -1+6t$

$\Rightarrow \quad t = x-1 \quad \Rightarrow \quad y = 2+5(x-1) = 5x-3$

or $\boxed{y = 5x-3}$ plane Π_1

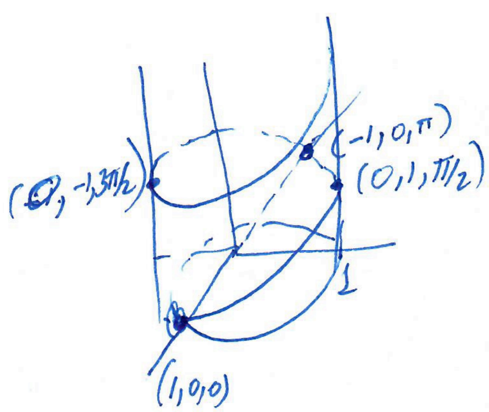
Also, $\quad z = -1+6(x-1) = 6x-7$

$\Rightarrow \quad \boxed{z = 6x-7}$ plane Π_2.

Example 4. Sketch the curve

$\vec{r}(t) = \langle \cos t, \sin t, t \rangle, \quad t \geq 0$

$x^2+y^2 = 1,$



$t=0: \langle 1,0,0 \rangle$

$t=\frac{\pi}{2}: \langle 0,1,\pi/2 \rangle$

$t=\pi: \langle -1,0,\pi \rangle$

$t=3\pi/2: \langle 0,-1,3\pi/2 \rangle$