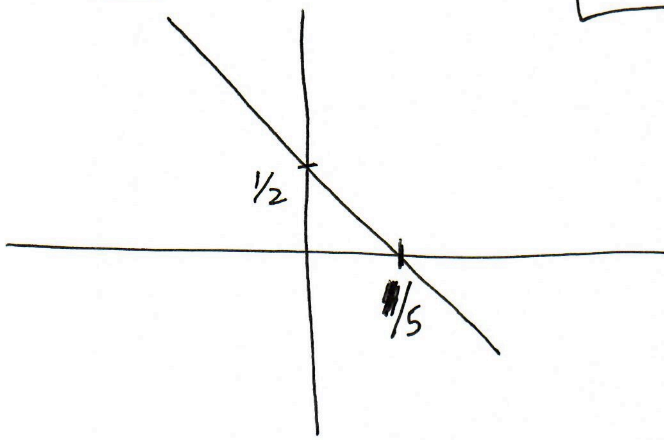
1.3

Summary of Class in Lines and Planes.

IN $\mathbb{R}^2$

1) General form: $ax + by = c$.

Example: $\boxed{5x + 2y = 1}$ or $\boxed{y = -\frac{5}{2}x + \frac{1}{2}}$



2) Vector form: $\begin{bmatrix} x \\ y \end{bmatrix} = t\begin{bmatrix} d_1 \\ d_2 \end{bmatrix} + \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}$

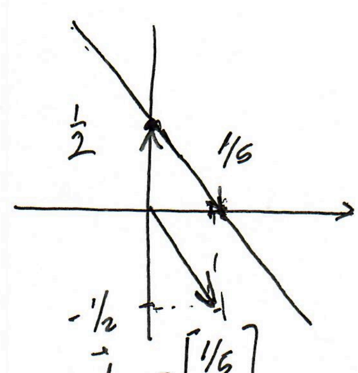
Example:
How do we get V.F. from G.F?
We need two points to obtain a direction Vector

x	y
0	1/2
1/5	0

$P(0, \frac{1}{2})$, $Q(\frac{1}{5}, 0)$

$\vec{d} = \vec{PQ} = \begin{bmatrix} 1/5 \\ -1/2 \end{bmatrix}$

Then, $\boxed{\begin{bmatrix} x \\ y \end{bmatrix} = t\begin{bmatrix} 1/5 \\ -1/2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/2 \end{bmatrix}}$



$\vec{d} = \begin{bmatrix} 1/5 \\ -1/2 \end{bmatrix}$

3) Parametric form: $x = x(t)$
$y = y(t)$

Example:
$$\boxed{\begin{aligned} x &= \tfrac{1}{5}t \\ y &= -\tfrac{1}{2}t + 1/2 . \end{aligned}} \qquad (2.1)$$

How do we go from parametric form to G.F?

Solve for t: $t = 5x$ Subst. in 2^{nd} equ.

$$y = -\tfrac{1}{2}(5x) + 1/2 = -\tfrac{5}{2}x + 1/2$$

$$\boxed{y = -5/2\, x + 1/2}$$

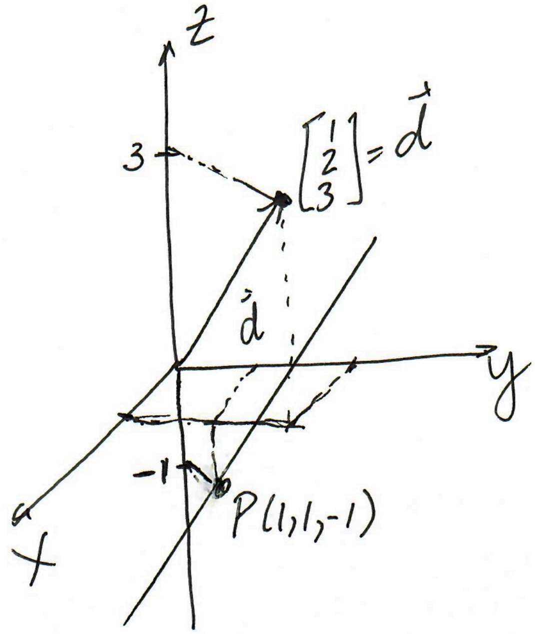
IN $\mathbb{R}^3$: The line is only represented in Vector form (or parametric form).

Two Cases: Given a direction Vector

$\vec{d} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and a point $P(1,1,-1)$

Vector Equation:
$$\boxed{\begin{bmatrix} x \\ y \\ z \end{bmatrix} = t \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}}, \qquad (2.2)$$

$$\boxed{\vec{x} = t\vec{d} + \vec{p}.} \qquad (2.3)$$



In parametric form: $\boxed{\begin{aligned} x &= t+1 \\ y &= 2t+1 \\ z &= 3t-1 \end{aligned}}$ (3.1)

<u>Second Case:</u> we can obtain (2.2)

from two points: $t=2$

$P(1,1,-1)$ and $Q(3,5,6)$

In fact, $\vec{d} = \vec{PQ} = \begin{bmatrix}2\\4\\6\end{bmatrix}$

<u>Vector equation:</u>

$$\begin{bmatrix}x\\y\\z\end{bmatrix} = t\begin{bmatrix}2\\4\\6\end{bmatrix} + \begin{bmatrix}1\\1\\-1\end{bmatrix}$$

Also

Planes:

General form: $\boxed{ax+by+cz=d}$

Example: $\boxed{2x+5y-z=6}$ Sect 1.3 # 13.

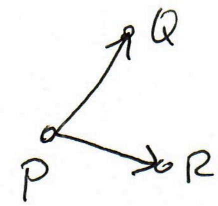
Vector form: Find three points in plane.

$x=1,\ y=1 \Rightarrow 2+5-z=6 \Rightarrow z=1 \Rightarrow P(1,1,1)$

$x=4,\ y=0 \Rightarrow 8-z=6 \Rightarrow z=2 \Rightarrow Q(4,0,2)$

$x=0,\ y=1 \Rightarrow 5-z=6 \Rightarrow z=-1 \Rightarrow R(0,1,-1)$

With these three points, we form the plane equation in vector form. In fact,



$$\vec{u}=\vec{PQ}=\begin{bmatrix}3\\-1\\1\end{bmatrix},\qquad \vec{v}=\vec{PR}=\begin{bmatrix}-1\\0\\2\end{bmatrix}$$

$$\boxed{\begin{bmatrix}x\\y\\z\end{bmatrix}=t\begin{bmatrix}3\\-1\\1\end{bmatrix}+s\begin{bmatrix}-1\\0\\2\end{bmatrix}+\begin{bmatrix}1\\1\\1\end{bmatrix}}$$

Parametric Form:

$$\boxed{\begin{aligned}x&=3t-s+1\\ y&=-t+1\\ z&=t+2s+1\end{aligned}}$$

Normal Form:

$$\boxed{\begin{array}{l} \vec{n}\cdot(\vec{x}-\vec{p}) = 0 \\ \text{or} \quad \vec{n}\cdot\vec{x} = \vec{n}\cdot\vec{p} \end{array}}$$

Example:

$$\vec{n} = \begin{bmatrix} 2 \\ 5 \\ -1 \end{bmatrix}, \quad P(1,1,1)$$

Then

$$\boxed{\begin{bmatrix} 2 \\ 5 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} x-1 \\ y-1 \\ z-1 \end{bmatrix} = 0} \Leftrightarrow$$

$$2x - 2 + 5y - 5 - z + 1 = 0$$

$$\boxed{2x + 5y - z = 6}$$