Worksheet
03/06/2020

Name: ________________________________

Let us compute approximately the integral $I = \int_1^3 x^2 dx$ by

- left-point rule (call the sum L_n),
- trapezoidal rule (call the sum T_n),

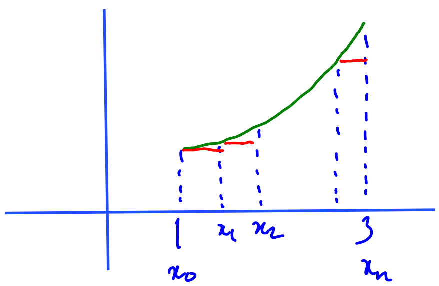
with $n+1$ equally spaced sample points $1 = x_0 < \ldots < x_n = 3$.

(a) Write L_n and T_n using sigma notation.

(b) Find n such that L_n approximates I with error not exceeding $\epsilon = 10^{-4}$.

(c) The same question as in Part (b) for T_n.

(a)



The width of each subinterval is

$$h = \frac{3-1}{n} = \frac{2}{n}.$$

We have

$$x_0 = 1,$$

$$x_1 = 1+h,$$

$$x_2 = 1+2h,$$

$$\ldots$$

$$x_n = 1+nh.$$

Thus, $x_k = 1 + kh = 1 + \frac{2k}{n}$.

The left-point Riemann sum is

$$L_n = hf(x_0) + \cdots + hf(x_{n-1}) = h\sum_{k=0}^{n-1} f(x_k)$$

$$= h\sum_{k=0}^{n-1} x_k^2$$

$$L_n = h\sum_{k=0}^{n-1}\left(1+\frac{2k}{n}\right)^2$$

$$T_n = \frac{1}{2}h\left(f(x_0)+f(x_1)\right) + \cdots + \frac{1}{2}h\left(f(x_{n-1})+f(x_n)\right)$$

$$= \frac{1}{2}h\sum_{k=0}^{n-1}\left(f(x_k)+f(x_{k+1})\right)$$

$$= \frac{1}{2}h\sum_{k=0}^{n-1}\left[\left(1+\frac{2k}{n}\right)^2 + \left(1+\frac{2(k+1)}{n}\right)^2\right]$$

We will do Part (b) and (c) next time.