Worksheet
3/4/2020

Name: ______________________________

1. Given an integer n and numbers α_j, β_j, γ_j for $j = 1, 2, \ldots, n-1$, how would you solve for M_1, M_2, …, M_n from the following system?

$$\begin{cases} \alpha_1 M_1 + \beta_1 M_2 & = & \gamma_1, \\ \alpha_2 M_2 + \beta_2 M_3 & = & \gamma_2 \\ \cdots & \cdots & \cdots \\ \alpha_{n-1} M_{n-1} + \beta_{n-1} M_n & = & \gamma_{n-1}. \end{cases}$$

Can you write an algorithm to solve it?

Because there are n unknowns and $n-1$ equations, we have the freedom to choose one unknown. Let us choose $M_1 = 0$. Then M_2 can be found from the first equation:

$$M_2 = \frac{\gamma_1 - \alpha_1 M_1}{\beta_1}.$$

Then M_3 can be found from the second equation

$$M_3 = \frac{\gamma_2 - \alpha_2 M_2}{\beta_2}$$

and so on. One can write the procedure as an algorithm as follows.

$M_1 = a$; (some specific number)

for $j = 2:n$

$$M_j = \frac{\gamma_{j-1} - \alpha_{j-1} M_{j-1}}{\beta_{j-1}}$$

end

* Matrix method :

One can solve the system using matrix method. With M_1 considered as being known, we can write the system as

$$\begin{cases} \beta_1 M_2 = \gamma_1 - \alpha_1 M_1, \\ \alpha_2 M_2 + \beta_2 M_3 = \gamma_2, \\ \cdots\cdots \\ \alpha_{n-1} M_{n-1} + \beta_{n-1} M_n = \gamma_{n-1} \end{cases}$$

In matrix form,

$$\underbrace{\begin{bmatrix} \beta_1 & 0 & \cdots & \cdots & 0 \\ \alpha_2 & \beta_2 & 0 & \cdots & 0 \\ 0 & \alpha_3 & \beta_3 & 0 \cdots & 0 \\ \vdots & & & & \\ 0 & 0 & \cdots & \alpha_{n-1} & \beta_{n-1} \end{bmatrix}}_{(n-1)\times(n-1)} \underbrace{\begin{bmatrix} M_2 \\ M_3 \\ \vdots \\ M_n \end{bmatrix}}_{(n-1)\times 1} = \underbrace{\begin{bmatrix} \gamma_1 - \alpha_1 M_1 \\ \gamma_2 \\ \vdots \\ \gamma_{n-1} \end{bmatrix}}_{(n-1)\times 1}$$

This is the form $AX = b$. The solution is $X = A^{-1}b$.

One can enter matrix A in Matlab by first initializing A as an $(n-1)\times(n-1)$ matrix whose entries are all zeros.

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A = zeros(n-1);
```

Then we fix the coefficients on the diagonal:

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for j = 1:n-1
        A(j,j) = βj,
end
```

Then we fix the coefficients on the sub-diagonal

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for j = 1:n-2
    A(j,j+1) = α_{j+1};
end
```

2. How would you plot the following functions on the interval $[0, 3]$ on the same graph using Matlab? (Use the command **hold on**)

$$\begin{aligned}
s_1(t) &= -t^2 + t \\
s_2(t) &= t^2 - 3t + 2 \\
s_3(t) &= t^2 - 5t + 6.
\end{aligned}$$

See Lecture 24 (toward the end).