Worksheet

1/17/2020

1. Find approximately the value of largest positive number and the smallest that can be represented precisely by the double precision floating-point format. How big is the dynamic range of this format?

$$x = \sigma \cdot \bar{x} \cdot 2^e$$

Only consider $\sigma = 1$.

x is biggest when $\bar{x} = (1.\underbrace{11\ldots1}_{52})_2$ and $e = 2046 - 1023 = 1023$

$$x_{max} = (1.\underbrace{11\ldots1}_{52})_2 \cdot 2^{1023} \approx 2 \cdot 2^{1023} = 2^{1024} \approx 8.988 \times 10^{307}$$

$$x_{min} = (0.0\ldots01)_2 \times 2^{-1022} = 2^{-52} \times 2^{-1022} = 2^{-1074}.$$

$$\text{Dynamic range} = \frac{x_{max}}{x_{min}} \approx \frac{2^{1024}}{2^{-1074}} = 2^{2098}.$$

2. Write number 1 in form of $\sigma \cdot \bar{x} \cdot 2^e$.

$$1 = 1 \cdot (1.00\ldots0)_2 \cdot 2^0$$

3. Let s be the smallest number greater than 1 that can be represented precisely by the double precision floating-point format. Write s in form of $\sigma \cdot \bar{x} \cdot 2^e$.

$$s = 1 \cdot (1.\underbrace{00\ldots0}_{51}1)_2 \cdot 2^0 = 1 + 2^{-52}$$

$$\approx \ldots$$

4. The difference $s - 1$ is called the machine ϵ of the floating-point format. How big is the machine ϵ of the double precision floating-point format?

$$\varepsilon = s - 1 = 2^{-52} \approx \ldots$$