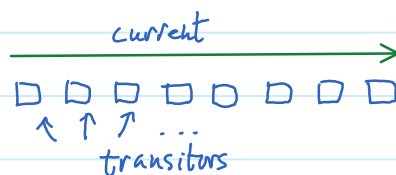


Lecture 5

Wednesday, January 15, 2020

Last time, we mentioned a type of error that comes from machine representation. To represent a number, one needs to specify a base system. A natural base to human beings is the decimal system. That is because we have 10 fingers. Counting all the fingers gives us numbers from 1 to 10. (In fact, the word "digit" also means finger or toe.) Counting all the fingers again gives us the teen-numbers. Counting again gives us the twenty-numbers, ...

For a computer, the decimal system is no longer natural. Computers run by electricity. They contain a vast number of so-called transistors, each



storing some charge. An electric current passing through these transistors can "turn on" or "off" certain transistors. Thus, each transistor can assume one of the two states (on or off), representing a digit in the binary system. Therefore, the binary system is more natural for computers to work with rather than the decimal system.

Let us recall some operations related to binary numbers: conversion from decimal to binary and vice versa, rounding, chopping, adding, multiplying.

Ex:

Convert 183 from decimal to binary system.

We keep dividing 183 by 2. The remainders are written on the right, the quotients on the left. We stop when the quotient is equal to 0. Then write all the remainders from bottom to top.

183	1
91	1
45	1
22	0
11	1
5	1
2	0
1	1
0	

$$183 = (10110111)_2$$

In terms of power of 2:

$$183 = 2^7 + 2^5 + 2^4 + 2^2 + 2^1 + 2^0$$

Ex: Convert 0.3 from decimal system to binary system.

$0.3 \times 2 = 0.6 \longrightarrow 0$	$\left. \begin{array}{l} \text{repeating} \end{array} \right\}$
$0.6 \times 2 = 1.2 \longrightarrow 1$	
$0.2 \times 2 = 0.4 \longrightarrow 0$	
$0.4 \times 2 = 0.8 \longrightarrow 0$	
$0.8 \times 2 = 1.6 \longrightarrow 1$	
$0.6 \times 2 = 1.2 \longrightarrow 1$	
$0.2 \times 2 = 0.4 \longrightarrow 0$	

Thus, $0.3 = (0.0100110011001\dots)_2$

Although 0.3 has only one digit after the decimal point, its representation in binary system has infinitely many digits after the binary point.

In terms of power of 2:

$$0.3 = 2^{-2} + 2^{-5} + 2^{-6} + 2^{-9} + \dots \quad (\text{Infinite series})$$

Ex: Convert 6.3 from decimal to binary.

$$6.3 = 6 + 0.3$$

The whole number part is converted the way we did in Example 1.

$$6 = (100)_2.$$

The fractional part is done using the method in Example 2.

$$0.3 = (0.\underbrace{01001}\underbrace{1001}\underbrace{1001}\dots)$$

Therefore,

$$6.3 = 6 + 0.3 = (100.\underbrace{01001}\underbrace{1001}\underbrace{1001}\dots)_2.$$

Ex: Consider the binary number $a = (0.0110101)_2$

- Round a to 4 digit after the binary point.

$$a \approx (0.0111)_2$$

because the digit after 1 (the 4th digit after the dot is 0)

- Round a to 5 digit after the binary point.

$$a \approx (0.01111)_2 \quad (\text{because the number after}$$

because the digit after 0 (the 5th digit after the dot is 1)

- Round a to 3 digits after the binary point

$$a \approx (0.011)_2 + (0.001)_2 = (0.100)_2.$$

- Chop a to 3 digit after the dot.

$$a \approx (0.011)_2$$

- Chop a to 5 digit after the dot.

$$a \approx (0.01110)_2.$$

Ex: Perform the multiplication $(1.11)_2 \times (1.11)_2$

The diagram shows the multiplication of $(1.11)_2 \times (1.11)_2$. The partial products are aligned as follows:

$$\begin{array}{r} 1.11 \\ \times 1.11 \\ \hline 111 \\ 1110 \\ 11100 \\ \hline 11.0001 \end{array}$$

Carry propagation is indicated by colored arrows and text:

- Red arrow: $1+1=10$: write 0, carry over 1
- Green arrow: $1+1+1+1 = (4)_{10} = 100$: write 0, carry 10
- Black arrow: $1+1+10 = (4)_{10} = 100$: write 0, carry 10
- Grey arrow: $1+10 = 11$

The final result is 11.0001 . The dot is placed before the fourth digit from the right.

[See more examples on the worksheet.]