# Homework 8

Due 03/13/2020

Denote $I = \int_0^1 \frac{1}{1+4x^2} dx$.

1. Find the exact value of $I$ (for example, by finding an antiderivative of the integrand).
2. For a generic positive integer $n$, we partition the interval $[0, 1]$ into $n$ equal subintervals $[x_0, x_1]$, $[x_1, x_2]$, $\ldots$, $[x_{n-1}, x_n]$. Denote by $L_n$, $R_n$, $M_n$, $T_n$ the Riemann sums corresponding to left-point, right-point, midpoint and trapezoid rule. Use sigma notation to write a formula for each $L_n$, $R_n$, $M_n$, $T_n$.
3. With the help of your calculator, compute $L_4$, $R_4$, $M_4$, $T_4$. Which of them is closest to $I$ ?
4. Write Matlab codes to compute $L_n$, $R_n$, $M_n$, $T_n$ when $n = 8$, 16, 32, 64.
5. Denote by $e_n^{(L)} = |L_n - I|$ the error term from left-point rule. We use similar notations for $e_n^{(R)}$, $e_n^{(M)}$, $e_n^{(T)}$. It is known that

$$e_n^{(L)}, e_n^{(R)} \le \frac{K(b-a)^2}{2n}, \quad e_n^{(M)} \le \frac{\tilde{K}(b-a)^3}{24n^2}, \quad e_n^{(T)} \le \frac{\tilde{K}(b-a)^3}{12n^2}$$

   where $K = \max_{[a,b]} |f'(x)|$ and $\tilde{K} = \max_{[a,b]} |f''(x)|$. Find $n$ such that the left-point rule gives an error not exceeding $\epsilon = 0.0001$. The same question for the right-point, midpoint, trapezoid rule.
   *Hint: you don't need to find the exact values of $K$ and $\tilde{K}$. An upper bound for each of them would be sufficient for this problem.*