Homework 6

Due 02/28/2020

1. Find a polynomial of degree ≤ 3 that fits the points $(2,1)$, $(1,0)$, $(3,-1)$, $(0,2)$ using the following methods. Convert the polynomial into the standard form $P(x) = ax^3 + bx^2 + cx + d$. (If you don't want to simplify the polynomial by hand, you can use the command **simplify** of Matlab.)

   (a) Solving a system of equations.
   Hint: after writing the system in matrix form $Ax = b$, you can use Matlab to solve this equation by using the command $x = inv(A) * b$.

   (b) Lagrange's formula.

   (c) Newton's formula.

2. Let f be a function such that $f(1) = 3$, $f(2) = 1$, $f(3) = 0$. Compute the divided difference $f[1,2,3]$.

3. In this problem, you can use the Matlab program posted on course website and Canvas (also given in the lecture) that computes the interpolation polynomial. We want to see how well a given function can be approximated by the interpolation polynomials. Let f be a function. We divide the the interval $[-0.6, 0.6]$ into subintervals of the same length $h = 0.02$. The gridpoints are $-0.6 = x_1 < x_2 < \ldots < x_{61} = 0.6$. Take $N = 61$ points (x_1, y_1),… (x_N, y_N) on the graph of f.

   (a) For $f(x) = \sin x$, plot the graph of the interpolation P on the interval $[-0.6, 0.6]$. Plot f and all of P on the same graph (for example, by using the command **hold on**). Does the interpolation polynomial approximate well the function f on the interval $[-0.6, 0.6]$?

   (b) The same questions as in Part (a) but for $f(x) = \frac{1}{1+x}$.

   (c) We know that the error between f and P is estimated by

   $$|f(x) - P(x)| \leq \frac{1}{n}\left(\frac{b-a}{n-1}\right)^n \max_{[a,b]} |f^{(n)}| \qquad (*)$$

   Let $f(x) = \frac{1}{1+x}$ and $[a,b] = [-0.6, 0.6]$. Use Stirling approximation $\frac{\sqrt[m]{m!}}{m} \approx \frac{1}{e}$ (for large m) to show that the right hand side of $(*)$ goes to infinity as $n \to \infty$.

4. You are recommended to do Matlab Practice 3 (posted on course website and Canvas) before starting this problem.
   Write a function in Matlab that does the following:

   - Input:
     - a function f,
     - an array x, i.e. a vector $x = (x_1, x_2, \ldots, x_n)$.
   - Output: the divided difference $f[x_1, x_2, \ldots, x_n]$.

   Test your function with $f(t) = \frac{1}{1+t^2}$ and $x = (1,2,3,4)$.