Homework 3

Due 02/03/2020

Use Matlab Practice 2 posted on Canvas and course website as a practice before you start doing this homework.

1. In approximation theory, there is an well-known result called Weierstrass theorem (1885). It says that: *given a continuous function f defined on an interval $[a, b]$ and a prescribed error ϵ, one can always approximate f by a polynomial on $[a, b]$ such that the error is under ϵ.* In this problem, we will find explicitly such a polynomial using Taylor polynomial (without invoking Weierstrass theorem).

   (a) Find a polynomial P such that

   $$\max_{x \in [2,4]} |\cos(x^2) - P(x)| < 10^{-3}.$$

   Hint: use the fact that $\cos(t) = 1 - \frac{t^2}{2!} + \frac{t^4}{4!} - \frac{t^6}{6!} + \ldots$

   (b) Plot function $f(x) = \cos(x^2)$ and function $P(x)$ which you found in Part (a) on the interval $[2, 4]$ on the same plot.
   Note: the graphs might be too close to each other to distinguish.

2. Consider the toy model of the IEEE double precision floating-point format as described in Homework 2. Perform the following operations on floating-point numbers. *Write your final answers in both floating-point format and decimal format.*

   (a) $(1.001)_2 \times 2^2 \ + \ (1.100)_2 \times 2^4$
   (b) $(0.010)_2 \times 2^{-6} \ + \ (1.001)_2 \times 2^2$
   (c) $(1.101)_2 \times 2^7 \ + \ (1.000)_2 \times 2^7$
   (d) $(0.001)_2 \times 2^{-3} \ \times \ (1.110)_2 \times 2^{-4}$
   What do you notice when adding two numbers of quite different sizes?

3. On an attempt to have Matlab compute the sum $S = 0.1 + 0.2 + \ldots + 0.9$, someone writes the following code:

```
s = 0
x = 0
while x~=1.0
    s = s + x
    x = x + 0.1
end
S = s
```

   (a) Test this code on Matlab. Why does the program keep running indefinitely?
   Note: to terminate the procedure, place the cursor in the command window and press Ctrl + C.

   (b) What should be changed in the code to make it stop?

4. On an attempt to have Matlab compute the sum $S = 1 + 2 + \ldots + 9$, a person writes the following code:

```
s = 0
x = 0
while x~=10
    s = s + x
    x = x + 1
end
S = s
```

(a) Test this code on Matlab. Does the program keep running indefinitely?

(b) What causes the difference compared to Problem 3?

5. In this problem, we will compute approximately a real root of the equation $x^3 - x^2 - 1 = 0$.

   (a) Graph the function $f(x) = x^3 - x^2 - 1$ on the interval $[a_0, b_0] = [0, 2]$.

   (b) Use the bisection method to find the interval $[a_4, b_4]$.

   (c) Approximate the root of $f(x) = 0$ with error not exceeding 10^{-2}.