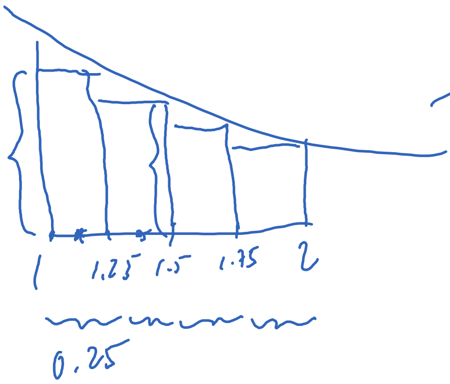
Review for Final exam

Friday, March 13, 2020 7:50 AM

$$f(x) = \frac{1}{(1+x^2)^2}$$



$$R_4 = 0.25 \times f(1.25) + 0.25 \times f(1.5) + 0.25 \times f(1.75) + 0.25 \times f(2).$$

$$= \ldots$$

$$M_4 = 0.25 \times f(1.125) + 0.25 f(1.375) + \ldots$$

(d). $|M_n - I| < 10^{-4}$

$$|M_n - I| \leq \frac{(b-a)^3}{24n^2} \max_{[1,2]} |f''|$$

$$\boxed{|M_n - I| \leq \frac{1}{24n^2} \max_{[1,2]} |f''| \qquad (*)}$$

Find upper bound for $|f''|$.

$$f(x) = \frac{1}{(1+x^2)^2}$$

$$f'(x) =$$

$$f''(x) = \frac{24x^2 - 4(1+x^2)}{(1+x^2)^4} = \frac{24x^2 - 4 - 4x^2}{(1+x^2)^4} = \frac{20x^2 - 4}{(1+x^2)^4}$$

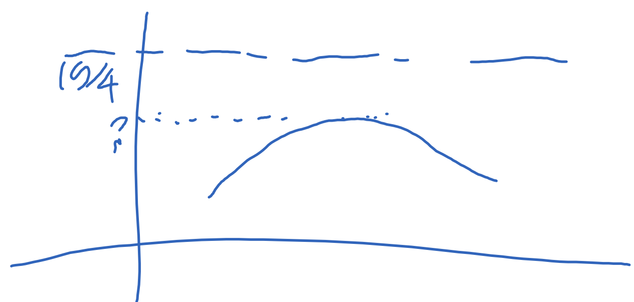
$$f''(x) = \frac{24x^2 - 4(1+x^2)}{(1+x^2)^4} = \frac{24x^2 - 4 - 4x^2}{(1+x^2)^4} = \frac{20x^2-4}{(1+x^2)^4}$$

$$|f''(x)| = \frac{20x^2-4}{(1+x^2)^4} \qquad x \in [1,2]$$

want to bound from above.

$$\underbrace{\frac{20x^2-4}{(1+x^2)^4}} \le \frac{20 \times 2^2 - 4}{(1+1^2)^4} = \frac{76}{16} = \frac{19}{4}$$

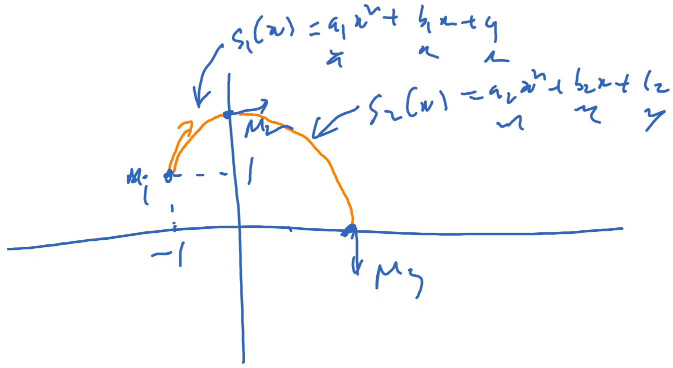
$\forall x \in [1,2]$



$$\max_{[1,2]} |f''| \le \frac{19}{4}$$

$$|M_n - I| \le \underbrace{\frac{1}{24n^2} \frac{19}{4}}$$

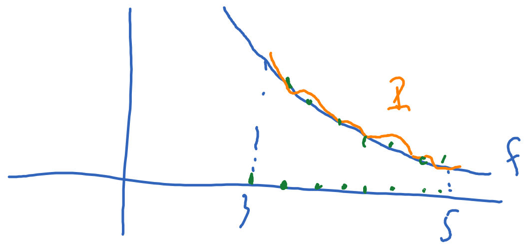
find n s.t. $\frac{1}{24n^2} \frac{19}{4} < 10^{-4}$



6)

$$|f(x) - P(x)| \leq \frac{1}{n}\left(\frac{5-3}{n-1}\right)^n \underbrace{\max_{[3,5]} |f^{(n)}|}_{??}$$

$$\boxed{f(x) = \frac{1}{x^2-1}}$$



$$f'(x) = \frac{-2x}{(x^2-1)^2}$$

$$f''(x) = \frac{-2(x^2-1)^2 - (-2x)(2)(2x)(x^2-1)}{(x^2-1)^4}$$

$$f(x) = \frac{A}{x+1} + \frac{B}{x-1}$$ (partial fraction decomposition).

$$A = \frac{-1}{2}, \quad B = \frac{1}{2}$$

$$f(x) = \left(-\frac{1}{2}\right)(x+1)^{-1} + \frac{1}{2}(x-1)^{-1}$$

$$f'(x) = \left(-\frac{1}{2}\right)(-1)(x+1)^{-2} + \frac{1}{2}(-1)(x-1)^{-2}$$

$$f''(x) = \left(\frac{-1}{2}\right)(-1)(-2)(x+1)^{-3} + \frac{1}{2}(-1)(-2)(x-1)^{-3}$$

$$f^{(n)}(x) = \left(-\frac{1}{2}\right)\underbrace{(-1)(-2)\cdots(-n)}(x+1)^{-n-1} +$$

$$\frac{1}{2}(-1)(-2)\cdots(-n)(x-1)^{-n-1}$$

$$= \left(-\frac{1}{2}\right)\underbrace{(-1)^n n!}(x+1)^{-n-1} + \underbrace{\frac{1}{2}}\underbrace{(-1)^n} n!(x-1)^{-n-1}$$

$$f^{(n)}(x) = \frac{1}{2}(-1)^n n! \left(-(x+1)^{-n-1} + (x-1)^{-n-1} \right)$$

$$|f^{(n)}(x)| = \frac{1}{2} n! \underbrace{\left| -(x+1)^{-n-1} + (x-1)^{-n-1} \right|}_{\text{how to bound this from above ?}} \qquad (*)$$

$$|a+b| \le |a| + |b| \quad (\text{triangle ineq})$$

$$\left| -(x+1)^{-n-1} + (x-1)^{-n-1} \right| \le (x+1)^{-n-1} + (x-1)^{-n-1}$$

$$\le 4^{-n-1} + 2^{-n-1}$$

$$f^{(n)}(x) \le \frac{1}{2} n! \left(4^{-n-1} + 2^{-n-1} \right)$$

???

$$|f(x) - p(x)| \le \frac{1}{n} \left(\frac{5-3}{n-1} \right)^n \max_{[3,5]} |f^{(n)}|$$

$$\le \underbrace{\frac{1}{n} \left(\frac{2}{n-1} \right)^n \frac{1}{2} n! \left(4^{-n-1} + 2^{-n-1} \right)}_{< 10^{-4}.}$$