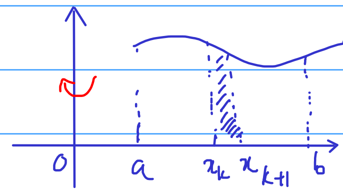
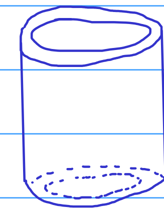
Lecture 11 (2/15/2019)

* Correction:





Radius of inner circle: x_k (not $x_k - a$)

" outer " : x_{k+1} (not $x_{k+1} - a$)

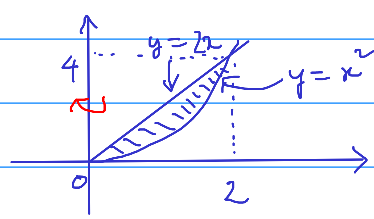
Consequently, the volume of the solid obtained by revolving about the y-axis the region R under the graph of f and above interval [a, b] is

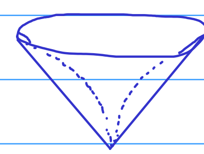
$$\text{vol} = \int_a^b \overbrace{\underbrace{2\pi x}_{\substack{\text{base} \\ \text{circumference}}} \underbrace{f(x)}_{\text{height}}}^{\text{lateral area of cylinder}} dx$$

If one rotates R about the axis $x = x_0 \leq a$ then the volume is

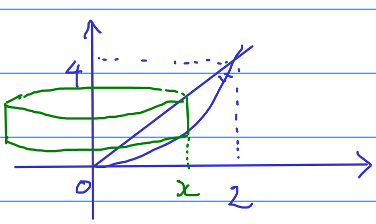
$$\text{vol} = \int_a^b 2\pi(x - x_0) f(x)\, dx$$

Ex





Volume by shells:

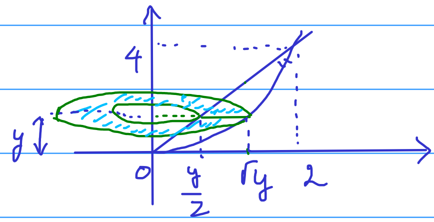


Radius of base : x

height : $2x - x^2$

$$\text{vol} = \int_0^2 \underbrace{\underbrace{2\pi x}_{\substack{\text{base} \\ \text{cir.}}} \underbrace{(2x - x^2)}_{\text{height}}}_{\text{lat. area of cylinder}} dx = \frac{8\pi}{3}$$

volume by slicing:

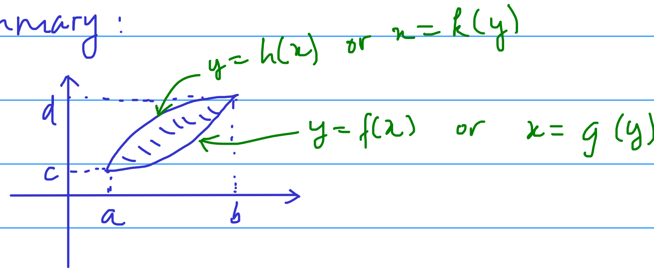


Area of slice at position y:

$$\pi\left((\sqrt{y})^2 - \left(\frac{y}{2}\right)^2\right) = \pi\left(y - \frac{y^2}{4}\right)$$

$$\text{vol} = \int_0^4 \pi\left(y - \frac{y^2}{4}\right)dy = \frac{8\pi}{3}$$

* Summary:



• Revolve about x-axis:

$$\text{vol} = \int_a^b \pi\left(h(x)^2 - k(x)^2\right)dx \quad \text{(slicing along } x\text{-direction)}$$

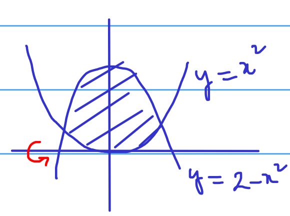
$$= \int_c^d 2\pi y\left(g(y) - k(y)\right)dy \quad \text{(shells chopped along } y\text{-direction)}$$

• Revolving about y-axis:

$$\text{vol} = \int_a^b 2\pi x\left(h(x) - f(x)\right)dx \quad \text{(shells chopped along } x\text{-dir.)}$$

$$= \int_c^d \pi\left(g(y)^2 - k(y)^2\right)dx \quad \text{(slicing along } y\text{-direction)}$$

Ex:



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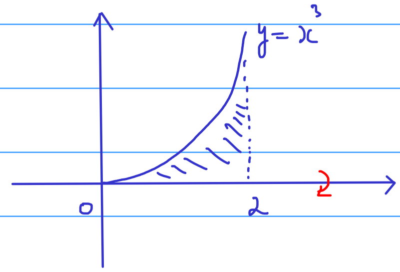

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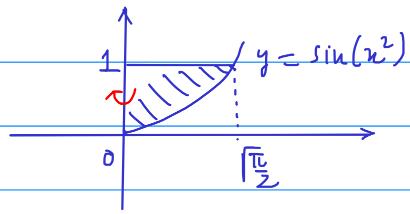

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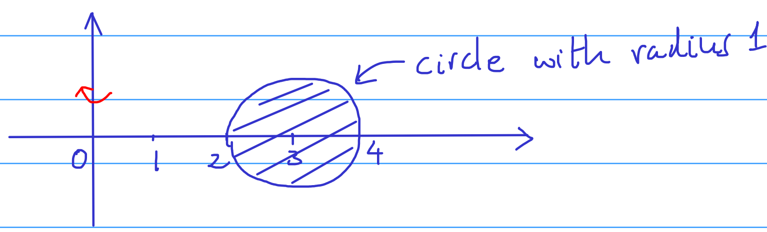

the solid is a torus