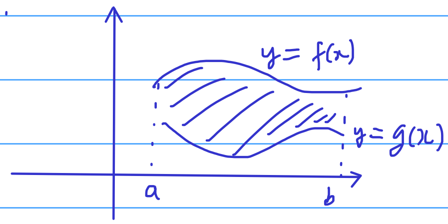
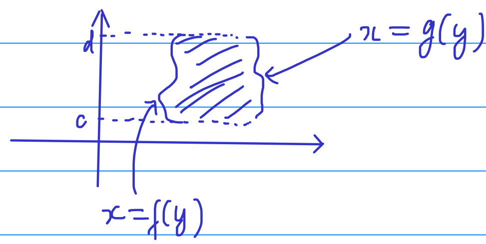
Lecture 10 (2/13/2019)

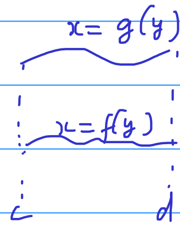
Recall:



$$\text{Area} = \int_a^b (f(x) - g(x))\,dx$$



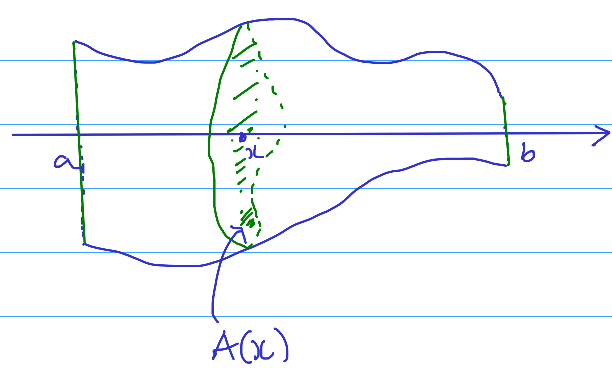
Turn this image 90° degree :



$$\text{Area} = \int_c^d (g(y) - f(y))\,dy$$

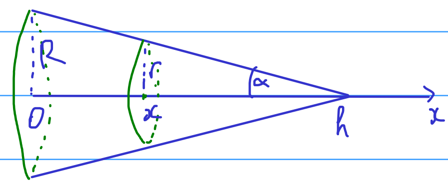
Recall: volume of a shape $= \displaystyle\int_a^b A(x)\,dx$ } this is called slicing method

$A(x)$: area of cross section at position x.



Ex:

Cone of heigh h and base radius R



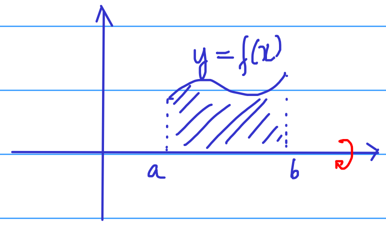
$$\text{volume} = \int_0^h \pi r^2 \, dx$$

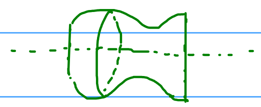
$$\tan\alpha = \frac{R}{h} = \frac{r}{h-x} \Rightarrow r = \frac{R}{h}(h-x)$$

$$A(x) = \pi r^2 = \frac{\pi R^2}{h^2}(h-x)^2$$

$$\text{vol} = \frac{\pi R^2}{h^2}\int_0^h (h-x)^2 \, dx = \frac{\pi R^2 h}{3}$$

Slicing method can be used to compute volume of a <u>solid of revolution</u>.

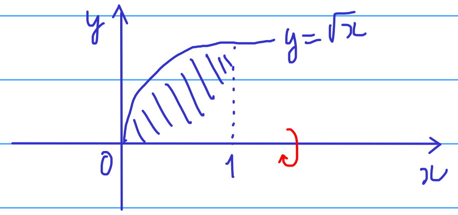


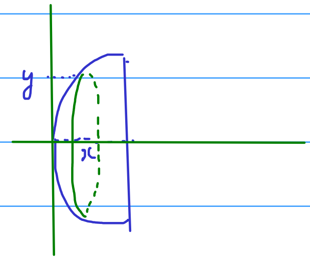


A lot of clay products (bowls, vases, glasses, ...) are solid of revolution because they are made in the process of revolving/spinning a wheel.

$$\text{Area} = \int_a^b A(x)\,dx = \int_a^b \pi f(x)^2 \, dx.$$

Ex:

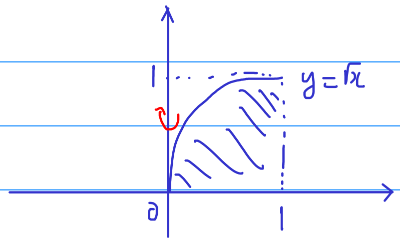




$$A(x) = (\sqrt{x})^2 \pi = \pi x$$

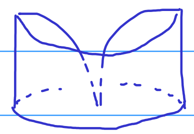
$$\text{vol} = \int_0^1 \pi x \, dx = \frac{\pi}{2}.$$

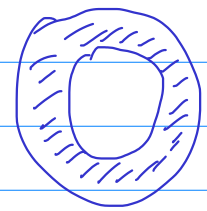
Ex :

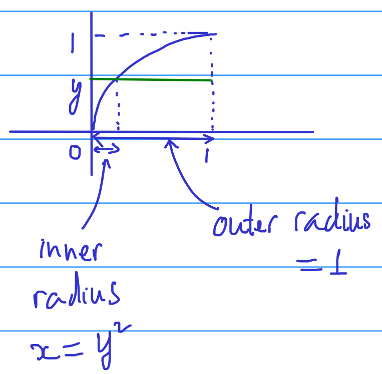


solid obtained by revolving the area about the y-axis.

Cross sections along the y-axis are annuli.





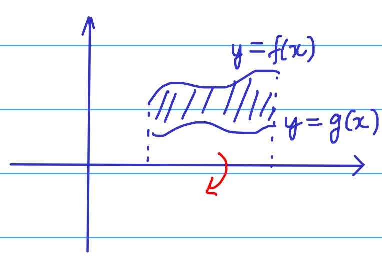


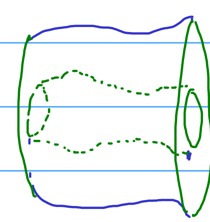
Area of cross section

$$A(y) = \pi\left(1^2-(y^2)^2\right) = \pi(1-y^4)$$

$$\text{Vol} = \int_0^1 \pi(1-y^4)\,dy$$

In general,



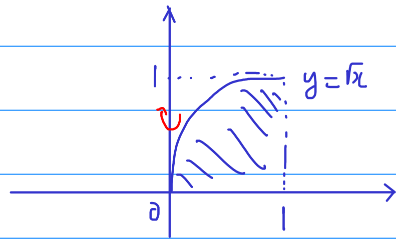


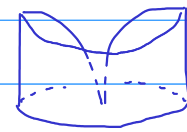
$$\text{Vol} = \int_a^b \pi\left(f(x)^2-g(x)^2\right)dx$$

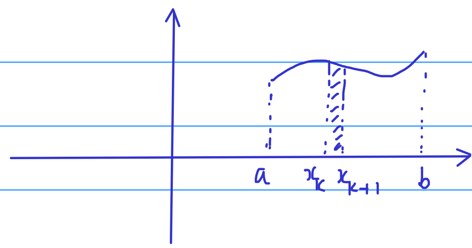
This is also called the <u>washer</u> method.

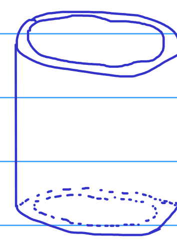
The case $g(x)\equiv 0$ is referred to as <u>disk</u> method.

Let's consider another method to compute the volume of a solid of revolution.









$a = x_0 < x_1 < x_2 < \cdots < x_n = b$

$$x_k^* = \frac{x_k + x_{k+1}}{2} \quad \text{(midpoint of the } k\text{'th subinterval)}$$

Volume of the outer cylinder : $\pi x_{k+1}^2 f(x_k^*)$

" inner " : $\pi x_k^2 f(x_k^*)$

Volume of the k'th shell : $\pi x_{k+1}^2 f(x_k^*) - \pi x_k^2 f(x_k^*)$

$$= \pi f(x_k^*) \left(x_{k+1}^2 - x_k^2\right)$$

$$= \pi f(x_k^*) (x_{k+1} + x_k)(x_{k+1} - x_k)$$

$$= 2\pi x_k^* f(x_k^*) (x_{k+1} - x_k)$$

Volume of the solid $\approx$ sum of volume of shells

$$= \sum_{k=0}^{n-1} 2\pi x_k^* f(x_k^*) (x_{k+1} - x_k)$$

This is a Riemann sum of the function $g(x) = 2\pi x f(x)$.

As $n \to \infty$:

$$\text{Vol} = \int_a^b g(x)\,dx = \int_a^b 2\pi x f(x)\,dx.$$