Some review problems for Midterm

1. Find the following indefinite integrals:

   (a) $\int \frac{2}{9+x^2}dx$

   (b) $\int \cos 2x \sin 3x dx$

   (c) $\int \frac{e^{1/x^2}}{x^3}dx$

   (d) $\int \frac{\cos\sqrt{x}}{\sqrt{x}}dx$

2. Find the following definite integrals:

   (a) $\int_0^1 \frac{3}{\sqrt{9-4x^2}}dx$

   (b) $\int_0^\pi \sin^2 3x dx$

   (c) $\int_0^{\pi/2} \sin^3 x \cos^2 x dx$

   (d) $\int_0^9 4\sqrt[3]{1-x}dx$

3. Do the following:

   (a) Find the intercepts of the curves $y = 9 - x^2$and $y = 8x$ in the first quadrant.

   (b) Find the intercepts of the curves $y = 9 - x^2$ and $y = 5x/2$ in the first quadrant.

   (c) Find the area of the region bounded by the curves $y = 9 - x^2$, $y = 5x/2$, $y = 8x$ in the first quadrant.

4. Find the length of the curve

$$y = \frac{x^{3/2}}{3} - x^{1/2} \quad \text{on} \quad [4,\ 16]$$

5. A roller coaster is running. Its height from the ground is measured as a function of time:

$$h(t) = t(\pi - t) + \frac{1}{2}\sin(6t)$$

   Compute the average height of the roller coaster from the ground during the time period $t \in [0, \pi]$. (Of course $h(t) \geq 0$ for all $t \in [0, \pi]$ for practical reason.)

6. For each integer $n \geq 1$, consider the sum

$$S_n = \sum_{k=0}^{n-1} \left(\frac{k^2}{n^2} + \frac{k}{n} + 1\right)\frac{3}{n}.$$

   Suppose we want to realize S_n as a Riemann sum of some function. That is to write S_n in the form

$$S_n = \sum_{k=0}^{n-1} f(x_k^*)(x_{k+1} - x_k).$$

   Follow the below steps:

(a) From the given formula of S_n, choose the width of each subinterval. (Answer is not unique.)

(b) With this choice of width, what is the height $f(x_k^*)$?

(c) Choose the initial point x_0. (Answer is not unique.)

(d) With x_0 and the subinterval width, compute the grid-points x_k in terms of k and n.

(e) What is the interval $[a, b]$?

(f) In the formula of S_n, the index k runs from 0 to $n-1$. This implies that the subintervals are labeled from 0 to $n-1$. What are the endpoints of the 0th, 1st, 2nd intervals? What are the endpoints of the k'th interval?

(g) Recall from the lecture that x_k^* denotes a sample point on the k'th subinterval. What is your choice for x_k^*? Is it a left point, right point, midpoint or neither of these?

(h) What is the function f?

7. With S_n given in the previous problem, compute $\lim_{n\to\infty} S_n$ using definite integral.