Solution to Problem 2 of HW 5

2a) $I = \int e^{-x} \cos x \, dx$

$u = \cos x \quad \ldots\ldots \quad du = -\sin x \, dx$

$dv = e^{-x} dx \quad \ldots\ldots \quad v = -e^{-x}$

$$I = -e^{-x} \cos x - \underbrace{\int e^{-x} \sin x \, dx}_{J} \qquad (*)$$

$u = \sin x \quad \ldots\ldots \quad du = \cos x \, dx$

$dv = e^{-x} dx \quad \ldots\ldots \quad v = -e^{-x}$

$$J = -e^{-x} \sin x + \int e^{-x} \cos x \, dx = -e^{-x} \sin x + I$$

Substitute J into $(*)$:

$$I = -e^{-x} \cos x - J = -e^{-x} \cos x - \left(-e^{-x} \sin x + I\right)$$

$$= -e^{-x} \cos x + e^{-x} \sin x - I$$

Then

$$2I = -e^{-x} \cos x + e^{-x} \sin x$$

$$I = -\frac{e^{-x} \cos x + e^{-x} \sin x}{2} + C$$

2b) $I = \int x^2 e^x \, dx$

$u = x^2 \quad \ldots\ldots \quad du = 2x \, dx$

$dv = e^x dx \quad \ldots\ldots \quad v = e^x$

$$I = x^2 e^x - \underbrace{\int 2x e^x \, dx}_{J} \qquad (*)$$

$u = 2x \quad \ldots\ldots \quad du = 2 \, dx$

$dv = e^x dx \quad \ldots\ldots \quad v = e^x$

$$J = 2xe^x - \int 2e^x dx = 2xe^x - 2e^x + C$$

Substitute J into $(*)$:

$$I = x^2e^x - (2xe^x - 2e^x + C)$$

$$= x^2e^x - 2xe^x + 2e^x - C$$

Conclusion:

$$\int x^2e^x dx = x^2e^x - 2xe^x + 2e^x + C$$