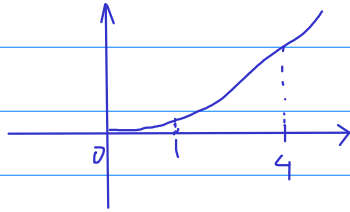


Solution to 3 of HW4

3a.



$$f(x) = x^{3/2}$$

$$f'(x) = \frac{3}{2} x^{1/2}$$

$$f'(x)^2 = \frac{9}{4} x$$

$$\sqrt{1 + f'(x)^2} = \sqrt{1 + \frac{81}{16} x}$$

Length of arc:

$$L = \int_1^4 \sqrt{1 + \frac{81}{16} x} dx$$

Let

$$u = 1 + \frac{81}{16} x$$

$$du = u' dx = \frac{81}{16} dx$$

$$\sqrt{1 + \frac{81}{16} x} dx = \sqrt{u} \frac{16}{81} du$$

x	1	4
u	$\frac{97}{16}$	$\frac{85}{4}$

$$L = \int_{97/16}^{85/4} \sqrt{u} \frac{16}{81} du = \frac{16}{81} \frac{2}{3} u^{3/2} \Big|_{97/16}^{85/4} = \dots$$

3b.

$$f(x) = x^3 + \frac{1}{12x}$$

$$f'(x) = 3x^2 - \frac{1}{12x^2}$$

$$f'(x)^2 = 9x^4 - 2(3x^2)\left(\frac{1}{12x^2}\right) + \frac{1}{144x^4} = 9x^4 - \frac{1}{2} + \frac{1}{144x^4}$$

$$1 + f'(x)^2 = 9x^4 + \frac{1}{2} + \frac{1}{144x^4}$$

$$= \left(3x^2 + \frac{1}{12x^2}\right)^2$$

$$\sqrt{1 + f'(x)^2} = 3x^2 + \frac{1}{12x^2}$$

$$\begin{aligned} \text{Length of arc} &= \int_1^2 \sqrt{1 + f'(x)^2} dx = \int_1^2 \left(3x^2 + \frac{1}{12x^2}\right) dx \\ &= \left(x^3 - \frac{1}{12x}\right) \Big|_1^2 \\ &= \left(2^3 - \frac{1}{12(2)}\right) - \left(1^3 - \frac{1}{12(1)}\right) \\ &= \frac{169}{24} \end{aligned}$$