Homework 2

1. Consider two following sums:

$$S_n = \sum_{k=0}^{n-1} \frac{k}{n^2}, \qquad S'_n = \sum_{k=1}^{n} \frac{k+2}{n^2}$$

   Given that $\lim_{n\to\infty} S_n = \frac{1}{2}$, find $\lim_{n\to\infty} S'_n$.
   (*Hint:* compute the difference $S'_n - S_n$.)

2. (Similar to Problems 21-24 of Section 5.2, page 359)
   Each of the following sums is an approximation of the area of some region. Given that this is a region between graph of a function and the x-axis. Determine the corresponding function f and the interval. Use Mathematica to sketch the region.

   (a)
$$\sum_{k=1}^{n} \left(1 + \frac{k^2}{n^2}\right) \frac{1}{n}$$

   (b)
$$\sum_{k=1}^{n} \left(1 + \frac{k+1}{n}\right) \frac{2}{n}$$

   (c)
$$\sum_{k=1}^{n} \ln\left(1 + \frac{k+\frac{2}{3}}{n}\right) \frac{1}{n}$$

3. Practice finding antiderivatives: answer the first 10 homework questions (out of 21) of Section 4.9 on MyMathLab.

4. Practice using geometry to compute definite integrals: do Problems 25, 27, 29 of Section 5.2, page 359. Use Mathematica to sketch the graph of the integrands.