2) (i) $$\sum_{k=1}^{n}\left(1+\frac{k^2}{n^2}\right)\frac{1}{n}$$

is a Riemann sum of function $f(x) = 1+x^2$, with sample points

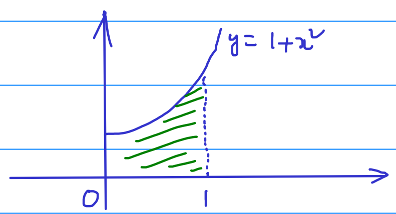
$$x_k^* = \frac{k}{n} \quad , \; 1 \le k \le n$$

These sample points are taken on interval $[0,1]$

$$0 \;\text{—+—+—+—+———+—+—}\; 1$$

Thus, the given Riemann sum is an approximation of

$$\int_0^1 (1+x^2)\,dx$$



(ii) $$I_n = \sum_{k=1}^{n}\left(1+\frac{k+1}{n}\right)\frac{2}{n}$$

This sum has the same limit (as $n\to\infty$) as the sum

$$J_n = \sum_{k=1}^{n}\left(1+\frac{k}{n}\right)\frac{2}{n}$$

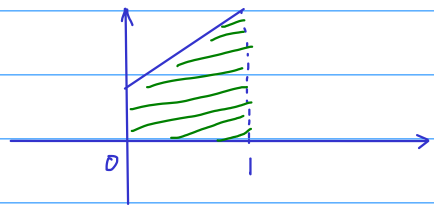
Indeed, the difference $$I_n - J_n = \sum_{k=1}^{n}\frac{1}{n}\,\frac{2}{n} = \frac{2}{n^2}\cdot n = \frac{2}{n} \to 0$$

$$J_n = \sum_{k=1}^{n}\left(2+\frac{2k}{n}\right)\frac{1}{n}$$

J_n is Riemann sum of function $f(x) = 2+2x$ with sample points $$x_k^* = \frac{k}{n} \quad , \; 1 \le k \le n$$

on the interval $[0,1]$. Thus, this sum approximates

$$\int_0^1 (2+2x)\,dx$$



(iii) The sum

$$I_n = \sum_{k=1}^{n} \ln\left(1+\frac{k+2/3}{n}\right)\frac{1}{n}$$

has the same limit (as $n\to\infty$) as the sum

$$J_n = \sum_{k=0}^{n-1} \ln\left(1+\frac{k+2/3}{n}\right)\frac{1}{n}$$

Indeed, the difference

$$I_n - J_n = \ln\left(1+\frac{n+2/3}{n}\right)\frac{1}{n} - \ln\left(1+\frac{0+2/3}{n}\right)\frac{1}{n}$$

$$= \frac{1}{n}\left[\ln\left(1+\frac{n+2/3}{n}\right) - \ln\left(1+\frac{2/3}{n}\right)\right]$$

$$= \frac{1}{n}\ln\frac{1+\frac{n+\frac{2}{3}}{n}}{1+\frac{2/3}{n}} \longrightarrow 0 \quad \text{as } n\to\infty$$

J_n is a Riemann sum of the function $f(x) = \ln(1+x)$ with sample points $x_k^* = \dfrac{k+2/3}{n}$, $0\le k\le n-1$

on the interval $[0,1]$. Thus, J_n (and thus I_n) approximates

$$\int_0^1 \ln(1+x)\,dx$$