Homework 1

1. Practice using summation notations: problems 29, 40, 41 (c, d, f, h) of Section 5.1 (page 344-345).

2. Use the identity $(k+1)^2 - k^2 = 2k+1$ to find an explicit formula for the sum $\sum_{k=1}^{n} k$ in terms of n.

3. Expand $(k+1)^3 - k^3$ and use it to find an explicit formula for the sum $\sum_{k=1}^{n} k^2$ in terms of n.

4. To each of the functions below, do the following:

   (i) Graph the function on the given interval. You may need to use Mathematica.

   (ii) Divide the given interval into n equal subintervals (n is a generic number). Then write down the corresponding Riemann sum, using the summation notation.

   (iii) Use Mathematica to compute the Riemann sum you got with $n = 5$, $n = 10$, $n = 100$.

   (a) $\frac{1}{1+x^2}$ on the interval $[0, 2]$. Use left-point rule.

   (b) $\ln x$ on the interval $[1, 4]$. Use right-point rule.

   (c) $\sqrt{x+1}$ on the interval $[0, 3]$. Use midpoint rule.

   (d) $\sin x$ on the interval $[0, \pi]$. Use trapezoid rule.

5. Consider function $f(x) = x^2$ on the interval $[2, 3]$. Suppose we want to compute the area under f.

   (i) Divide interval $[2, 3]$ into n equal subintervals. Then write the Riemann sum using left-point rule.

   (ii) Use the formula you obtain in Problem 3 to simplify this sum.

   (iii) Find the (exact) area under the graph of f and above the interval $[2, 3]$.

6. With the function $f(x) = x^2$ as in Problem 5, we now want to compute the length of the graph of f over the interval $x \in [2, 3]$.

   (i) Write a sum that approximates the length of the graph of f.

   (ii) Find the approximate lengths for $n = 5$, $n = 10$, $n = 30$.