54 points in total

Group work 6

Due 3/13/2019

Name: ______________________________

Instructions: Show your work. Circle your final answers. The assignment has 8 pages.

1. Find the following indefinite integral:

$$\int \frac{\ln x}{x^2} dx$$

Hint: use integration by parts.

4pt

$u = \ln x \quad \ldots\ldots \quad du = \frac{1}{x} dx$

$dv = \frac{dx}{x^2} \quad \ldots\ldots \quad v = -\frac{1}{x}$

$$\int u\,dv = -\frac{1}{x}\ln x - \int -\frac{1}{x}\cdot\frac{1}{x}\,dx = -\frac{1}{x}\ln x + \int \frac{1}{x^2}dx$$

$$= -\frac{1}{x}\ln x - \frac{1}{x} + C$$

2. Evaluate the improper integral

$$\int_1^\infty \frac{\ln x}{x^2}dx = \lim_{b\to\infty} \int_1^b \frac{\ln x}{x^2}dx$$

4pt

$$\int_1^b \frac{\ln x}{x^2}dx = \left(-\frac{1}{x}\ln x - \frac{1}{x}\right)\Big|_1^b = -\frac{\ln b}{b} - \frac{1}{b} - \left(-\frac{1}{1}\ln 1 - \frac{1}{1}\right)$$

$$= \underbrace{-\frac{\ln b}{b}}_{\to 0} \underbrace{-\frac{1}{b}}_{\to 0} + 1 \longrightarrow 1 \quad \text{as } b\to\infty$$

3. To evaluate the integral

$$\int \frac{2x^3+x^2-6x+7}{x^2+x-6}dx$$

one follows the below steps.

(a) Do long division.

$$\begin{array}{r|l} & \quad 2x-1 \\ \hline x^2+x-6 & 2x^3+x^2-6x+7 \\ & -\;(2x^3+2x^2-12x) \\ \hline & \qquad -x^2+6x+7 \\ & \qquad -\;(-x^2-x+6) \\ \hline & \qquad\qquad 7x+1 \end{array}$$

4pt

$$\frac{2x^3+x^2-6x+7}{x^2+x-6} = 2x-1 + \frac{7x+1}{x^2+x-6}$$

(b) Do partial fraction decomposition for the fraction of remainder/divisor.

$$\frac{7x+1}{x^2+x-6} = \frac{7x+1}{(x-2)(x+3)} = \frac{A}{x-2} + \frac{B}{x+3}$$

4pt

want: $7x+1 = A(x+3) + B(x-2)$

Plug $x=-3$: $-20 = -5B \rightsquigarrow B=4$

Plug $x=2$: $15 = 5A \rightsquigarrow A=3$

$$\frac{7x+1}{x^2+x-6} = \frac{3}{x-2} + \frac{4}{x+3}$$

(c) Find the given integral by finding the integral of each term.

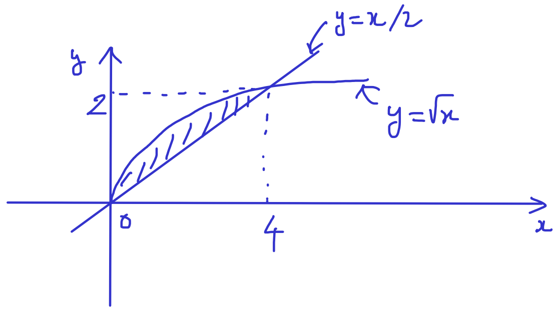
$$\int \cdots = \int \left(2x-1+\frac{3}{x-2} + \frac{4}{x+3}\right) dx$$

4pt

$$= x^2 - x + 3\ln|x-2| + 4\ln|x+3| + C$$

4. To compute the volume of the solid obtained by rotating the region bounded by the curves $y = \sqrt{x}$ and $y = \frac{x}{2}$ about the x-axis, one follows the below steps:

(a) Graph these two curves on the same graph. Then shade the region bounded by them.



3pt

(b) Find the points of intersection between these curves.

$\sqrt{x} = \frac{x}{2} \rightsquigarrow x = \frac{x^2}{4}$

$4x = x^2$

$x(4-x) = 0$

$x = 0$ or $x = 4$

3pt

For $x = 0$: $y = 0$

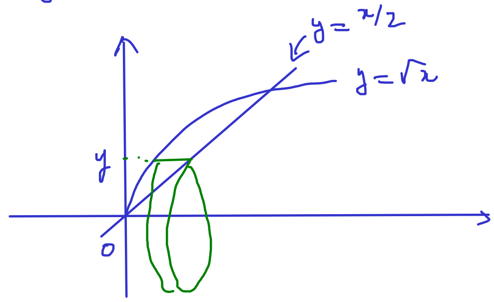
For $x = 4$: $y = 2$

$(0,0)$ and $(4,2)$

(c) Let S be the solid obtained by rotating the shaded region about the x-axis. First, specify which method you want to use to compute the volume of S (slice method or shell method). Then write the definite integral expressing the volume of S. Don't evaluate yet.

Say, use shell method:

3 pt



$$\int_0^2 \underbrace{2\pi y}_{\text{base circ.}} \underbrace{(2y - y^2)}_{\text{height}}\, dy$$

$y=\sqrt{x} \Rightarrow x = y^2$

$y = \frac{x}{2} \Rightarrow x = 2y$

(d) Evaluate the integral you have found above.

3pt

$$= 2\pi \int_0^2 (2y^2 - y^3)\, dy$$

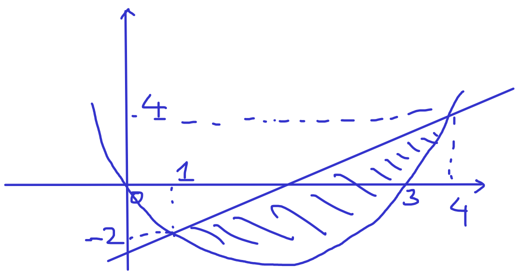
$$= 2\pi \left(\frac{2}{3} y^3 - \frac{y^4}{4} \right) \Big|_0^2$$

$$= 2\pi \left(\frac{16}{3} - \frac{16}{4} \right) = \frac{8\pi}{3}$$

5. To compute the volume of the solid obtained by rotating the region bounded by the curves $y = 2x - 4$ and $y = x^2 - 3x$ about the y-axis, one follows the below steps:

(a) Graph these two curves on the same graph. Then shade the region bounded by them.

3pt



(b) Find the points of intersection between these curves.

$x^2 - 3x = 2x - 4 \rightsquigarrow x^2 - 5x + 4 = 0 \rightsquigarrow x = 1$ or $x = 4$

For $x = 1$: $y = -2$

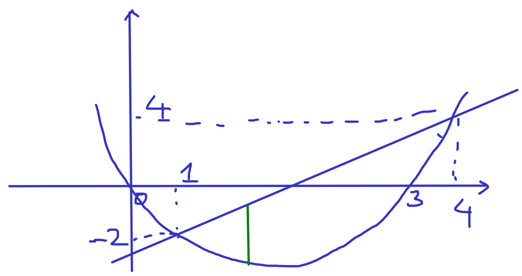
For $x = 4$: $y = 4$

3pt

$(1, -2)$ and $(4, 4)$

(c) Let S be the solid obtained by rotating the shaded region about the y-axis. First, specify which method you want to use to compute the volume of S (slice method or shell method). Then write the definite integral expressing the volume of S. Don't evaluate yet.

3pt



Say, use shell method.

$$\int_1^4 2\pi x\left(2x-4-(x^2-3x)\right)dx$$

(d) Evaluate the integral you have found above.

3pt

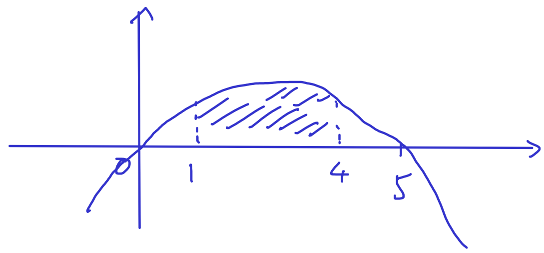
$$\int_1^4 2\pi x\left(-x^2+5x-4\right)dx$$

$$=2\pi\int_1^4\left(-x^3+5x^2-4x\right)dx$$

$$=2\pi\left(-\frac{x^4}{4}+\frac{5}{3}x^3-2x^2\right)\Big|_1^4=\frac{45\pi}{2}$$

6. To find the area of the surface generated when the curve $y = \sqrt{5x - x^2}$ on $[1, 4]$ is revolved about the x-axis, one follows the below steps:

(a) Sketch the graph of the given curve.



3pt

(b) Express the surface integral as a definite integral. Don't evaluate yet.

$$\int_1^4 2\pi f(x) \sqrt{1 + f'(x)^2}\, dx$$

where $f(x) = \sqrt{5x - x^2}$

3pt

(c) Evaluate the integral you have found above.

$$f'(x) = \frac{5-2x}{2\sqrt{5x-x^2}}$$

$$1+f'(x)^2 = 1+\frac{(5-2x)^2}{4(5x-x^2)} = \frac{4(5x-x^2)+(5-2x)^2}{4(5x-x^2)}$$

$$= \frac{20x-4x^2+(25-20x+4x^2)}{4(5x-x^2)}$$

4pt

$$= \frac{25}{4(5x-x^2)}$$

$$2\pi f(x)\sqrt{1+f'(x)^2} = 2\pi\sqrt{5x-x^2}\,\frac{5}{2\sqrt{5x-x^2}} = 5\pi$$

$$\int_1^4 \cdots\, dx = \int_1^4 5\pi\, dx = 15\pi$$