

Group work 2

Due 1/25/2019

Total: 48 points

Name: _____

Instructions: Show your work. In problems that require Mathematica, write one or two commands that best illustrate how you get the answer. You are not required to write all the codes. Circle your final answers. The assignment has 4 pages.

1. Find an antiderivative of:

(i) $\sin(2x)$

$$-\frac{1}{2} \cos(2x)$$

3pt

(ii) $\cos(2x) + x^2$

$$\frac{1}{2} \sin(2x) + \frac{x^3}{3}$$

3

2. Let F be an antiderivative of $f(x) = x^2 + e^x$ such that $F(0) = 2$. Determine F .

$$F(x) = \frac{x^3}{3} + e^x + C$$

4

Plug $x=0$:

$$F(0) = \frac{0^3}{3} + e^0 + C = 1 + C$$

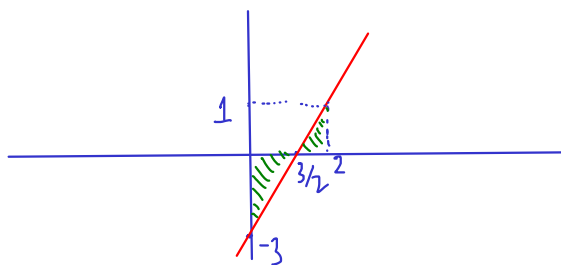
From the fact that $F(0)=2$, we get $C=1$.

Conclusion:

$$F(x) = \frac{x^3}{3} + e^x + 1$$

3. In this problem, we will use geometry to compute the definite integral $I = \int_0^2 (2x - 3)dx$.

(i) Graph the function $f(x) = 2x - 3$ over the interval $x \in [0, 2]$.



(ii) The given integral is the signed area of a region. Shade this region on the graph. 2

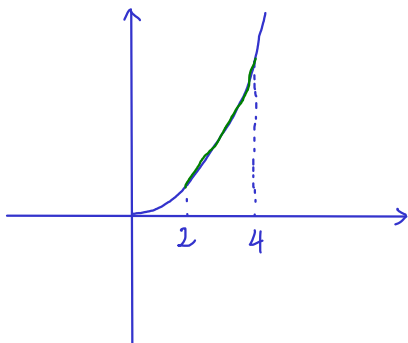
(iii) Compute the exact value of I .

$I = -\text{area of lower triangle} + \text{area of upper triangle}$

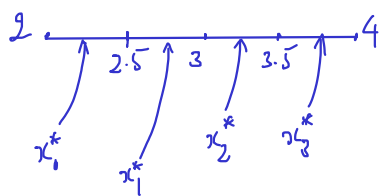
$$= -\frac{1}{2} \underset{\substack{\uparrow \\ \text{base}}}{3} \underset{\substack{\uparrow \\ \text{altitude}}}{\left(\frac{3}{2}\right)} + \frac{1}{2} \underset{\substack{\uparrow \\ \text{base}}}{\left(\frac{1}{2}\right)} \underset{\substack{\uparrow \\ \text{altitude}}}{1} = -\frac{9}{4} + \frac{1}{4} = -2$$

4. In this problem, we will use midpoint Riemann sum to approximate the definite integral $I = \int_2^4 x^2 dx$.

(i) Graph the function $f(x)$ on the interval $x \in [2, 4]$.



- (ii) Partition $[2,4]$ into 4 equal subintervals $[x_0, x_1]$, $[x_1, x_2]$, $[x_2, x_3]$, $[x_3, x_4]$. Determine x_0 , x_1 , x_2 , x_3 , x_4 .



$$\begin{aligned}x_0 &= 2 \\x_1 &= 2.5 \\x_2 &= 3 \\x_3 &= 3.5 \\x_4 &= 4\end{aligned}$$

3

- (iii) Compute the midpoint Riemann sum.

$$\text{Midpoints: } x_0^* = 2.25, \quad x_1^* = 2.75, \quad x_2^* = 3.25, \quad x_3^* = 3.75$$

$$\begin{aligned}\text{R-sum} &= f(x_0^*) \cdot 0.5 + f(x_1^*) \cdot 0.5 + f(x_2^*) \cdot 0.5 + f(x_3^*) \cdot 0.5 \\&= (2.25)^2 \cdot 0.5 + (2.75)^2 \cdot 0.5 + (3.25)^2 \cdot 0.5 + (3.75)^2 \cdot 0.5 \\&= 18.625\end{aligned}$$

4

5. In this problem, we will use right-point Riemann sum to approximate the definite integral $I = \int_2^4 x^2 dx$.

- (i) Partition $[2,4]$ into 100 equal subintervals $[x_0, x_1]$, $[x_1, x_2]$, $\dots$, $[x_{99}, x_{100}]$. What is the width of each subinterval?

$$\frac{4-2}{100} = \frac{1}{50} = 0.02$$

3

- (ii) Determine $x_0, x_1, x_2, \dots, x_{100}$.

$$\begin{aligned}x_0 &= 2 \\x_1 &= 2.02 \\x_2 &= 2.04 \\&\dots \\x_{100} &= 4\end{aligned}$$

$$x_k = 2 + (0.02)k \quad \text{or} \quad 2 + \frac{k}{50}$$

3

- (iii) Write the right-point Riemann sum using the sigma summation notation.

$$\sum_{k=0}^{99} f(x_{k+1}) \cdot 0.02 = \sum_{k=0}^{99} x_{k+1}^2 \cdot 0.02 = \sum_{k=0}^{99} \left(2 + \frac{k+1}{50}\right)^2 \cdot 0.02$$

3

(iv) Use Mathematica to compute this sum.

$$\text{Sum} \left[\left(2 + (k+1)/50 \right)^2 \cdot 0.02, \{k, 0, 99\} \right]$$

$$18.7868$$

3

6. Fill in the blanks to get a correct equation:

$$\sum_{k=1}^{100} k = 1 + 2 + 100 + \sum_{k=3}^{99} k$$

3

7. Fill in the blanks to get a correct equation:

$$\sum_{k=1}^n \sin \left(\frac{k\pi}{n} \right) = \sin \left(\frac{\pi}{n} \right) + \sum_{k=2}^n \sin \left(\frac{k\pi}{n} \right)$$

3

8. Given that the formula

$$1 + 2 + \dots + m = \frac{m(m+1)}{2}$$

is true for all integer m . Find an explicit formula of the following sum in terms of n .

$$\sum_{k=2}^{2n-1} \frac{k}{n^2} = \frac{1}{n^2} \sum_{k=2}^{2n-1} k$$

4

$$= \frac{1}{n^2} (2 + 3 + 4 + \dots + (2n-1))$$

$$= \frac{1}{n^2} (-1 + 1 + 2 + 3 + 4 + \dots + (2n-1))$$

sum of the first
 $2n-1$ positive integers.

$$= \frac{1}{n^2} (-1 + n(2n-1))$$

$$= \frac{2n^2 - n - 1}{n^2}$$

take $m = 2n-1$:

$$1 + 2 + 3 + \dots + (2n-1) = \frac{(2n-1)(2n-1+1)}{2}$$

$$= n(2n-1)$$