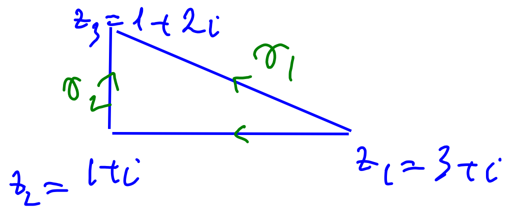
Worksheet

5/18/2020

With $z_1 = 3+i$, $z_2 = 1+i$, $z_3 = 1+2i$, let γ_1 be the line segment from z_1 to z_3, and γ_2 be the path consisting of the line segment from z_1 to z_2 and the line segment from z_2 to z_3. Find the following integrals:

(a) $\int_{\gamma_1} z dz$



γ_1 is a line segment that passes through two points z_1 and z_3.

Thus,
$$y = \frac{y_3 - y_1}{x_3 - x_1}(x - x_1) + y_1 = \frac{2-1}{1-3}(x-3) + 1$$
$$= -\frac{1}{2}x + \frac{5}{2}.$$

Parametric equation of γ_1 is
$$x = -t$$
$$y = \frac{1}{2}t + \frac{5}{2}.$$

~~(b) $\int_{\gamma_2} z dz$~~

The complex parametrization of γ_1 is
$$\gamma_1(t) = x + it = -t + i\left(\frac{1}{2}t + \frac{5}{2}\right).$$
where $-3 \le t \le -1$.

We have
$$\gamma_1'(t) = -1 + i\left(\frac{1}{2} + \frac{5}{2}\right) = -1 + 3i$$

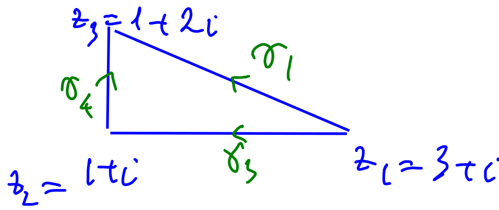
$$\int_{\underbrace{\gamma_1}_{f(z)=z}} z\, dz = \int_{-3}^{-1} \underbrace{\gamma_1(t)}_{f(\gamma_1(t))}\, \gamma_1'(t)\, dt = \int_{-3}^{-1} \left[-t + i\left(\frac{1}{2}t + \frac{5}{2}\right)\right](-1+3i)\, dt$$

$= \ldots$ (multiply, then split the integral to two parts: real and imaginary.)

(b) γ_2 Consists of two parts :

$$\gamma_3(t) = 3 - t + i \qquad 0 \le t \le 2$$

$$\gamma_4(t) = 1 + it \qquad 1 \le t \le 2.$$



$$\gamma_3'(t) = -1$$

$$\gamma_4'(t) = i$$

$$\int_{\gamma_2} z\,dz = \int_{\gamma_3} z\,dz + \int_{\gamma_4} z\,dz$$

$$= \int_0^2 \gamma_3(t)\,\gamma_3'(t)\,dt + \int_1^2 \gamma_4(t)\,\gamma_4'(t)\,dt$$

$$= \int_0^2 (3 - t + i)(-1)\,dt + \int_1^2 (1 + it)\,i\,dt$$

$$= \; \cdots\cdots$$

(c) $\int_{\gamma_1} \bar{z} dz$

(d) $\int_{\gamma_2} \bar{z} dz$