Worksheet
4/22/2020

Find the following limits. Distinguish between that the limit being equal to ∞ and that the limit does not exist.

(a) $\lim_{z\to i} \frac{z}{z+1}$

Let z_n be a sequence converging to i. Then

$$\frac{z_n}{z_n+1} \to \frac{i}{i+1} = \frac{i(1-i)}{2} = \frac{1}{2} + i\frac{1}{2} \quad \text{as } n\to\infty.$$

Conclusion:

$$\lim_{z\to i} \frac{z}{z+1} = \frac{1}{2} + i\frac{1}{2}.$$

(b) $\lim_{z\to\infty} \frac{z+i}{iz+1}$

$$\frac{z+i}{iz+1} = \frac{1+\frac{i}{z}}{i+\frac{1}{z}}$$

Because $z\to\infty$, we have $\frac{i}{z}\to 0$ and $\frac{1}{z}\to 0$. Thus,

$$\lim_{z\to\infty} \frac{z+i}{iz+1} = \frac{1}{i} = -i.$$

(c) $\lim_{z\to\infty} \frac{z+1}{z^2+1}$

(d) $\lim_{z\to 0} \frac{|z|^2}{z}$

(e) $\lim_{z\to\infty} \sin z$