Lecture 6

Friday, April 10, 2020 8:23 AM

$$ax^2+bx+c=0$$

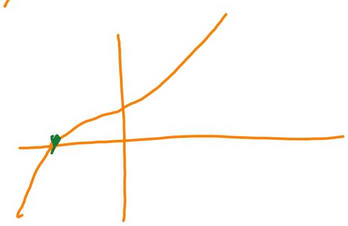
$$x^2+\frac{b}{a}x+\frac{c}{a}=0$$

$$\left(x+\frac{b}{2a}\right)^2+\frac{c}{a}-\left(\frac{b}{2a}\right)^2=0$$

$$x=\frac{-b\pm\sqrt{\Delta}}{2a}$$

$$\boxed{\underbrace{x^3+2x+2}_{f(x)}=0} \quad ??$$

$$f'(x)=3x^2+2>0$$



x_0 ... is a root

$$x^3+2x+2=(x-x_0)(x^2+\dots)$$

Cardano's method.

$$x^3+px+q=0$$

$$x=u+v$$

$$\text{LHS}=u^3+v^3+(3uv+p)(u+v)+q$$

$$\begin{cases} 3uv+p=0 \\ u^3+v^3+q=0 \end{cases}$$

$$\rightsquigarrow \begin{cases} uv=-p/3 \\ u^3+v^3=-q \end{cases} \rightsquigarrow \begin{cases} u^3v^3=-\frac{p^3}{27} \\ u^3+v^3=-q \end{cases}$$

$t=u^3$

$s=$

$$\rightsquigarrow \begin{cases} ts=-p^3/27 \\ t+s=-q \end{cases} \rightsquigarrow t, s \text{ solve}$$

$$w^2+qw-\frac{p^3}{27}=0$$

$$\Delta=q^2+\frac{4p^3}{27}$$

$$\begin{cases} ts = \alpha \\ t+s = \beta \end{cases} \longrightarrow t, s \text{ solve } w^2 - \beta w + \alpha = 0$$

$$t, s = \frac{1}{2}\left(-q \pm \sqrt{\Delta}\right)$$

$$\left.\Delta = q^2 + \frac{4p^3}{27}\right\} \quad \Delta > 0$$

$$u^3, v^3 = \frac{1}{2}\left(-q \pm \sqrt{\Delta}\right)$$

$$u = \sqrt[3]{\frac{1}{2}\left(-q + \underbrace{\sqrt{\Delta}}_{i\sqrt{3}}\right)} \quad u_1, u_2, u_3$$

$$v = \sqrt[3]{\frac{1}{2}\left(-q - \underbrace{\sqrt{\Delta}}_{i\sqrt{3}}\right)} \quad v_1, v_2, v_3$$

$$\Delta = -3 = 3i^2$$

$$\sqrt{\Delta} = \pm i\sqrt{3}$$

$$\boxed{x^3 + 3x^2 + 1 = 0}$$

$$x = u + v = \cdots$$

$u_1 + v_1$ $\quad$ $u_1 + v_2$ $\quad$ $u_2 + v_3$ $\quad \cdots$

$u_2 + v_2$ $\quad$ $u_1 + v_3$ $\quad \cdots \quad \cdots$

$$ax^3 + bx^2 + cx + d = 0$$

$$x^3 + \frac{b}{a}x^2 + \frac{c}{a}x + \frac{d}{a} = 0$$

$$y = x + \frac{b}{3a} \rightsquigarrow x = y - \frac{b}{3a}$$

$$\underbrace{\left(y - \frac{b}{3a}\right)^3}_{-3x\frac{b}{3a}xy^2} + \frac{b}{a}\underbrace{\left(y - \frac{b}{3a}\right)^2}_{\frac{b}{a}y^2} + \frac{c}{a}\left(y - \frac{b}{3a}\right) + \frac{d}{a} = 0$$

1. $z^3+3z^2+1=0$

$x = z+\frac{3}{3} = z+1$

$z = x-1$

$\underbrace{(x-1)^3}_{x^3-\cancel{3x^2}+3x-1} + \underbrace{3(x-1)^2}_{\cancel{3x^2}-6x+3} + 1 = 0$

$x^3-3x+3=0$

$p=-3$

$q=3$

$\to \Delta$

$\to u, v$

$az^3+bz^2+cz+d=0$

$z^3+\frac{b}{a}z^2+\frac{c}{a}z+\frac{d}{a}=0$

$x = z+\frac{b}{3a}$

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quadratic

cubic

standard form $a+bi$

polar $re^{i\theta}$

$e^x = 1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots$

$e=e^1$

$e^{ix} = \cos x + i\sin x = \operatorname{cis}(x)$

$e^{a+ib} = e^a e^{ib}$

$e^z = e^{a+ib} = ??$

$e^{u+v} = e^u e^v$

$$\boxed{e^{x+y} = e^x e^y} \longleftarrow \text{pure algebraic}$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$e^y = 1 + \frac{y}{1!} + \frac{y^2}{2!} + \frac{y^3}{3!} + \cdots$$

$$e^x e^y = \left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots\right)\left(1 + \frac{y}{1!} + \frac{y^2}{2!} + \cdots\right)$$

$$= 1 + (x+y) + \left(\frac{x^2}{2} + \frac{y^2}{2} + xy\right) + \cdots$$

$$= 1 + \frac{x+y}{1!} + \underbrace{\frac{x^2+y^2+2xy}{2}}_{\frac{(x+y)^2}{2!}} + \underset{\frac{(x+y)^3}{3!}}{\cdots} \cdots$$

$$e^a = 1 + \frac{a}{1!} + \frac{a^2}{2!} + \cdots$$

$$e^{ib} = 1 + \frac{ib}{1!} + \frac{(ib)^2}{2!} + \cdots$$

$$\left(1 + \frac{a}{1!} + \frac{a^2}{2!} + \cdots\right)\left(1 + \frac{ib}{1!} + \frac{(ib)^2}{2!} + \cdots\right)$$

$$\left.= 1 + (a+ib) + \frac{(a+ib)^2}{2!} + \cdots\right\} = e^{a+ib}$$

$$e^{a+ib} = e^a e^{ib} \longrightarrow \text{comes from the def.}$$

$$e^z = 1 + \frac{z}{1!} + \frac{z^2}{2!} + \frac{z^3}{3!} + \cdots$$

$$e^z = \underbrace{e^a} e^{ib}$$

$$e^a = |e^z|$$

$$b + k2\pi = \arg(e^z)$$

$$e^{ix} = \cos x + i\sin x$$

$$e^{-ix} = \cos(-x) + i\sin(-x)$$

$$e^{-ix} = \cos x - i\sin x$$

$$e^{ix} + e^{-ix} = 2\cos x$$

$$\begin{cases} \cos x = \dfrac{e^{ix} + e^{-ix}}{2} \\ \sin x = \dfrac{e^{ix} - e^{-ix}}{2i} \end{cases}$$

$$\cos z = \frac{e^{iz} + e^{-iz}}{2}$$

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i}$$