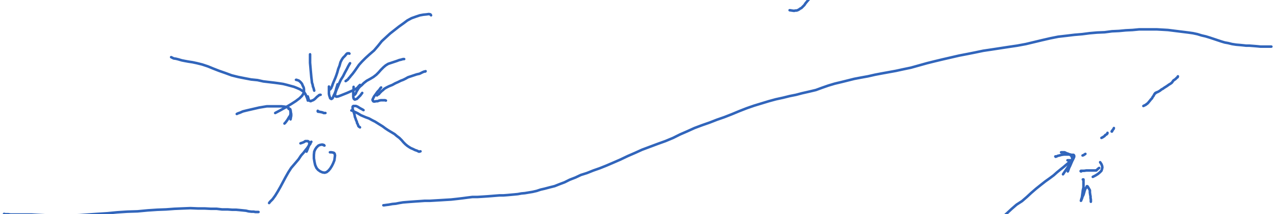
Lecture 21

Monday, May 18, 2020 8:59 AM

$$f'(z_0) = \lim_{h \to 0} \frac{f(z+h) - f(z)}{h}$$

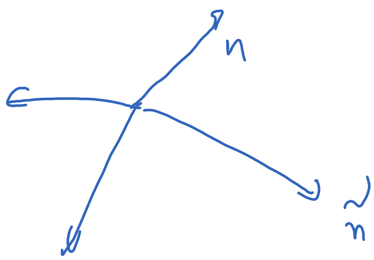


vector calculus 254

$f(x, y)$

$D_n f(x,y)$ (x, y)

$$= \lim_{t \to 0} \frac{f((x,y) + t\vec{n}) - f(x,y)}{t}$$



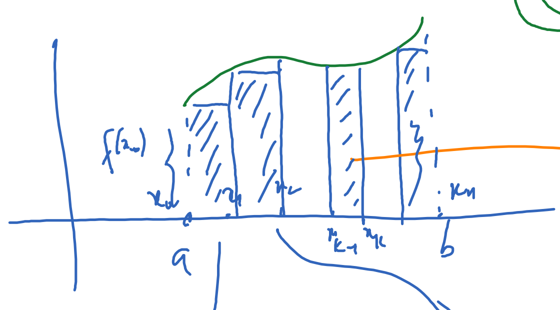
Integral of complex function?

review

$f : [a, b] \to \mathbb{R}$

$$\int_a^b f(x)\,dx = \lim_{n \to \infty} \sum_{k=1}^{n} f(x_{k-1})(x_k - x_{k-1})$$

$\sum$ $f(x_{k-1})$ $x_k - x_{k-1}$

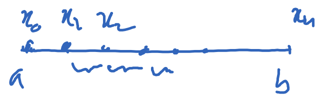


$$\sum_{k=1}^{n} f(x_{k-1})(x_k - x_{k-1})$$

$f(x_0)(x_1 - x_0)$ $f(x_1)(x_2 - x_1)$

$f : [a,b] \to \mathbb{C}$

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \underbrace{\sum_{k=1}^{n} \boxed{f(x_{k-1})}\,(x_k - x_{k-1})}_{I_n \in \mathbb{C}}$$



$f(x) = u(x) + i v(x)$

$$f(t) = \underbrace{u(t)}_{\in\mathbb{R}} + i \underbrace{v(t)}_{\in\mathbb{R}} \qquad t \in \mathbb{R}$$

$$\int_a^b f(t)\,dt = \lim_{n\to\infty} \underbrace{\sum_{k=1}^{n} f(t_{k-1})\,(t_k - t_{k-1})}$$

$$= \lim_{n\to\infty} \underbrace{\sum_{k=0}^{n-1} \underbrace{f(t_k)}_{u(t_k)+iv(t_k)}(t_{k+1} - t_k)}$$

$$\underbrace{\sum_{k=0}^{n-1} u(t_k)(t_{k+1} - t_k)} + i \underbrace{\sum_{k=0}^{n-1} v(t_k)(t_{k+1} - t_k)}$$

$$\xrightarrow{n\to\infty} \int_a^b u(t)\,dt \qquad\qquad \xrightarrow{n\to\infty} \int_a^b v(t)\,dt$$

If $f : [a,b] \to \mathbb{C}$ and $f = u + iv$ then

$$\int_a^b f(t)\,dt = \int_a^b u(t)\,dt + i \int_a^b v(t)\,dt$$

$$\boxed{\begin{aligned} &\int (u + iv)\,dt \\ &= \int u\,dt + i \int v\,dt \end{aligned}}$$

$f : \mathbb{C} \to \mathbb{C}$

$f : \mathbb{R} \to \mathbb{R}$

$\int_a^b f(x)\,dx$

$f : D \subset \mathbb{C} \longrightarrow \mathbb{C}.$

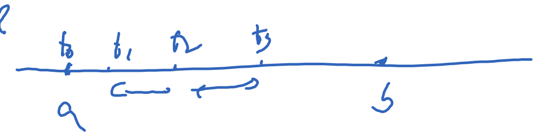
$$\int_\gamma f(z)\, dz$$

$f(z_{k-1})$

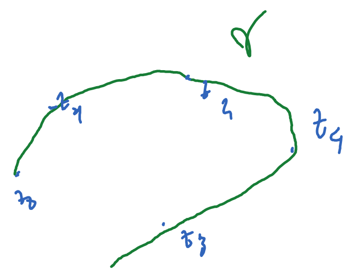
$z_k - z_{k-1}$

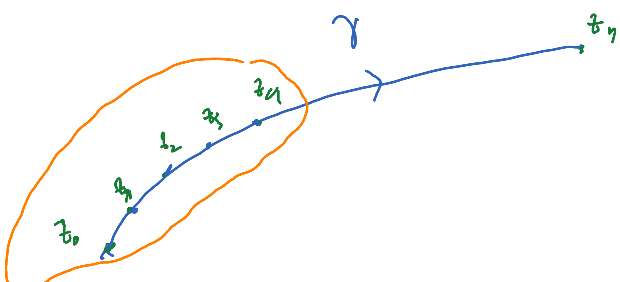
$z_0, z_1, z_2, \ldots, z_n$

$t \in \mathbb{R}$



$$\lim_{n\to\infty} \sum_{k=1}^{n} \underbrace{f(z_{k-1})}_{\in \mathbb{C}} \underbrace{(z_k - z_{k-1})}_{\in \mathbb{C}}$$





$\gamma(t)$

$$\int_\gamma f(z)\,dz = \lim_{n\to\infty} \Big(\underbrace{f(z_0)(z_1 - z_0) + f(z_1)(z_2 - z_1) + \cdots + f(z_{n-1})(z_n - z_{n-1})}_{\sum_{k=1}^{n} f(z_{k-1})(z_k - z_{k-1})} \Big)$$

How do we sample points on a curve?

$$\int_\gamma f(z)\,dz = \lim_{n\to\infty} \sum_{k=0}^{n-1} f(z_k)(z_{k+1}-z_k)$$

$$= \lim_{n\to\infty} \sum f(\gamma(t_k)) \underbrace{(\gamma(t_{k+1}) - \gamma(t_k))}_{\underbrace{\frac{\gamma(t_{k+1})-\gamma(t_k)}{t_{k+1}-t_k}}_{\approx \gamma'(t_k)}(t_{k+1}-t_k)}$$

$$\sum \approx \underbrace{\sum_k f(\gamma(t_k))\,\gamma'(t_k)\,(t_{k+1}-t_k)}$$

$$\Big\downarrow{\scriptstyle n\to\infty}$$

$$\int_\gamma f(z)\,dz = \int_a^b f(\gamma(t))\,\gamma'(t)\,dt$$

Ex: $\gamma(t) = t + it^2 \qquad \underline{0 \le t \le 1}$

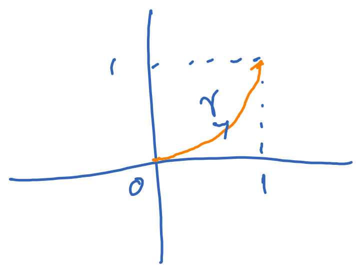
$\gamma'(t) = 1 + 2it$

$$\int_\gamma z\,dz = \int_0^1 (t+it^2)(1+2it)\,dt$$

$f(\gamma(t)) = f(t+it^2)$

$= t+it^2$

$f(z) = z$



$$\int_\gamma z\,dz = \int_0^1 \underbrace{(t+it^2)(1+2it)}\,dt = \int_0^1 (t-2t^3+i\,3t^2)\,dt$$

$t - 2t^3 + i(t^2 + 2t^2)$

$= t - 2t^3 + i(3t^2)$

$$= \underbrace{\int_0^1 (t-2t^3)\,dt} + i\underbrace{\int_0^1 3t^2\,dt}$$

$= \;?\; + i\;?$