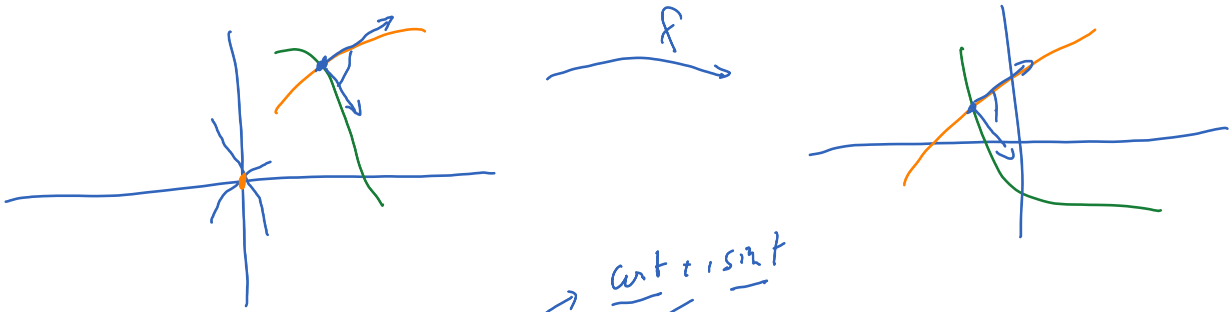
Lecture 20

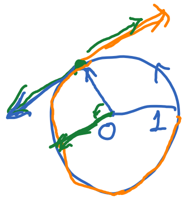
Friday, May 15, 2020 8:59 AM

$f(z) = z^2$ $\quad \underbrace{\text{holo. on } \mathbb{C}}_{\text{entire}}$ $\quad$ conformal on $\mathbb{C}\setminus\{0\}$.

$\mathbb{C} \xrightarrow{f} \mathbb{C}$

$f'(z) = 2z$





$\gamma(t) = e^{it}$ $\quad \underline{\cos t} + i\,\underline{\sin t}$ $\quad 0 \le t \le 2\pi$

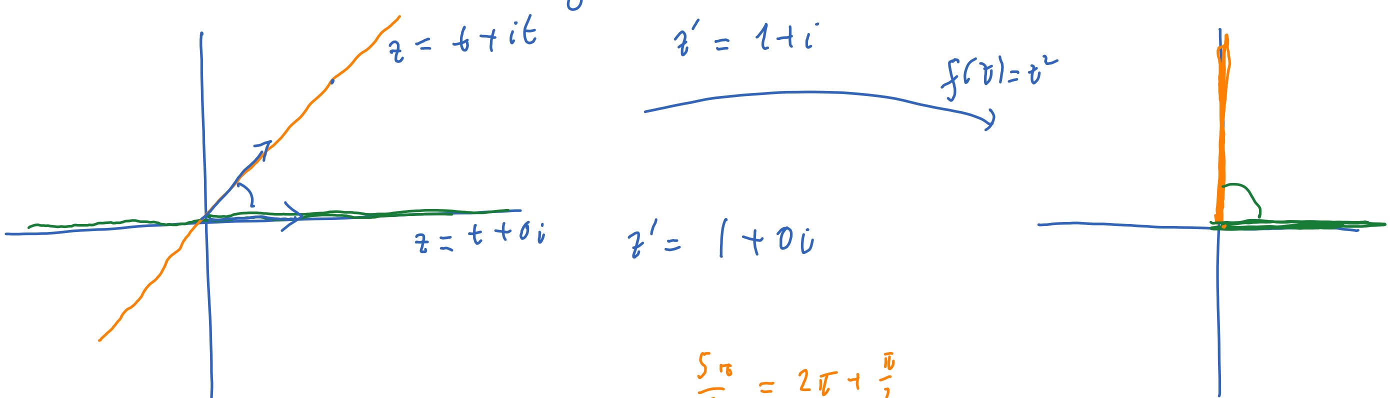
$\gamma'(t) = i e^{it}$

$= i\gamma(t)$

$= e^{i\frac{\pi}{2}}\gamma(t)$

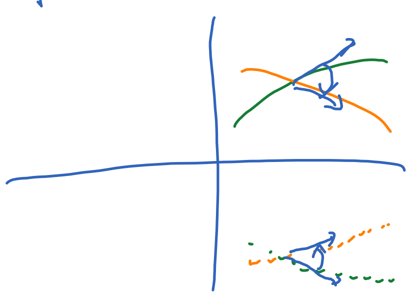
$\gamma_1(t) = e^{-it} \qquad 0 \le t \le 2\pi$

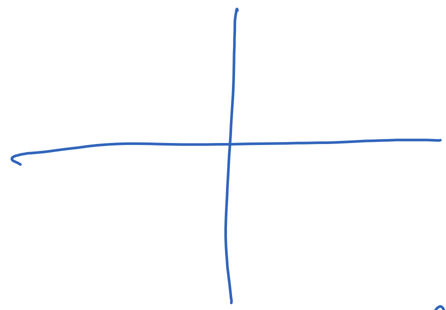
$\gamma_2(t) = \underbrace{t}_{x} + i\underbrace{\sqrt{1-t^2}}_{y}$



$\frac{5\pi}{2} = 2\pi + \frac{\pi}{2}$

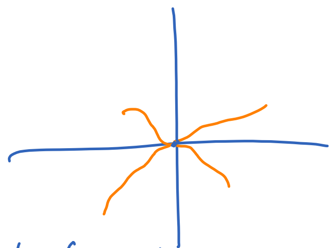
$f(z) = \bar{z}$ nowhere diff

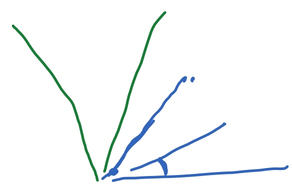




Thm f is conformal on D iff $\begin{cases} f \text{ is holo on } D \\ f'(z) \neq 0 \text{ on } D \end{cases}$

$$f(z) = \frac{z^2}{z-3i} \sim \frac{z^2}{0-3i} = \frac{-1}{3i} z^2$$





$\begin{cases} \text{int. by part} \\ \text{substitution} \end{cases}$

$$\left[\int u \underbrace{dv}_{v'dz} = uv - \int v\,du \right] \text{ int. by part}$$

$$\left[\int f(u)\,u'\,dz = \int f(u)\,du \right] \text{ subs.}$$

$$\int z \operatorname{Log} z\,dz \sim \int x \ln x\,dx$$

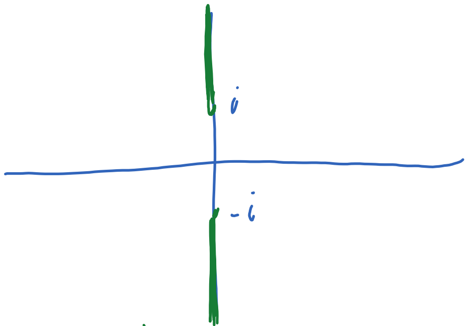
$du = z\,dz \rightsquigarrow u = \frac{z^2}{2}$

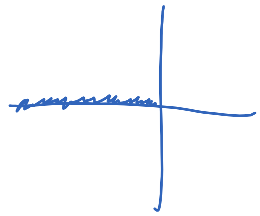
$v = \operatorname{Log} z \qquad dv = \frac{1}{z}\,dz$

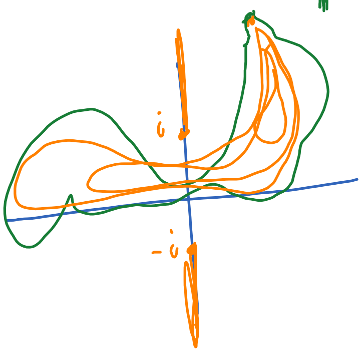
$$f(z) = \frac{z}{z^2+1} \rightsquigarrow \text{Log}(z^2+1)$$

$u = z^2+1$

$$z \xrightarrow[\text{poly}]{z^2+1} z^2+1 \xrightarrow[\text{Log}]{} \text{Log}(z^2+1)$$







f is cont on $\mathbb{C}\setminus\{\pm i\}$

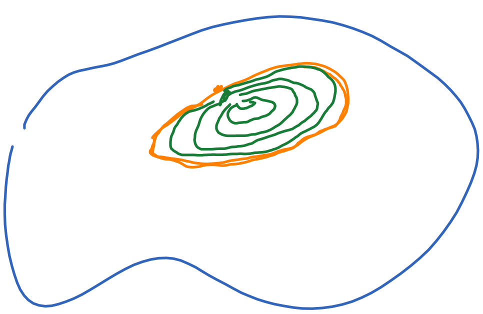
Claim: f doesn't have antider on $\mathbb{C}\setminus\{\pm i\}$

$\longrightarrow F(x) = \int_a^x f(t)\,dt$

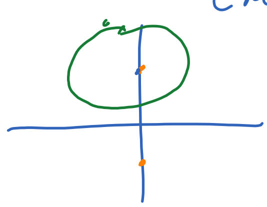
$F' = f$

$\mathbb{C}\setminus\{\text{two line}\}$ is simply connected

$\mathbb{C}\setminus\{\pm i\}$ is not simply connected



D is simply connected if any closed curve (loop) can be continuously contracted to a point in D

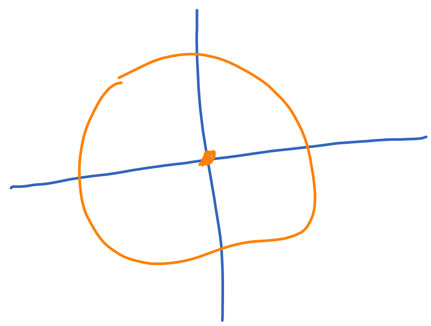


$\mathbb{C}\setminus\{\pm i\}$

$\boxed{\text{Thm: If } f \text{ is holo on } D \text{ and } D \text{ is simply connected then } f \text{ has an antiderivative on } D.}$

$f(z) = \frac{1}{z^2}$

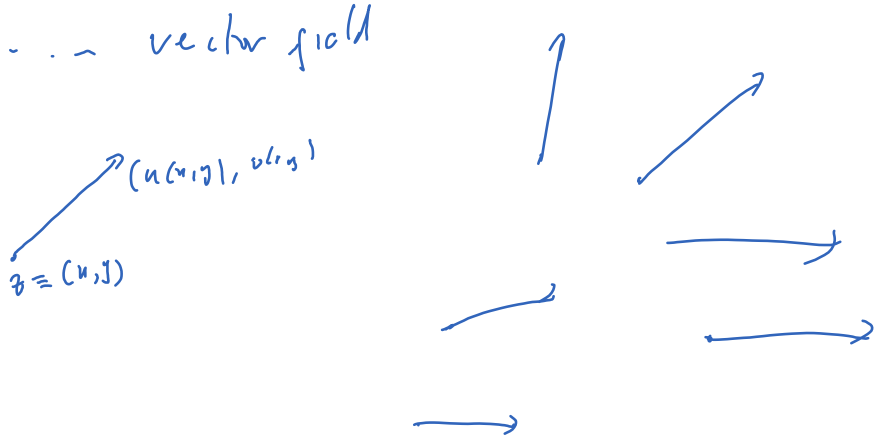
$\mathbb{C} \setminus \{0\}$ not simply conn



$F(z) = -\frac{1}{z}$

$f(z) = u(x,y) + i\, v(x,y)$

f ... vector field



$f(z) = z = x + iy$

$u(x,y) = x$

$v(x,y) = y$