Lecture 11

Wednesday, April 22, 2020 8:58 AM

e^z $\quad$ $\log z \rightsquigarrow \operatorname{Log} z$ $\quad$ k

$\sin z$ $\quad$ $\arcsin z \rightsquigarrow \operatorname{Arcsin} z$

$\cos z$ $\quad$ k, l $\quad$ no k, l

log $\quad$ $\sqrt{\ }$

$\pm\sqrt{2}$

$\tan w = z \longrightarrow w = \arctan z$

$\cos z \longrightarrow \arccos z$

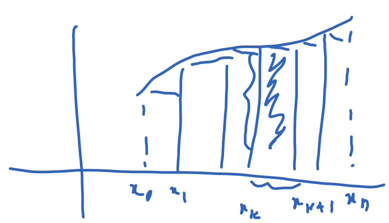
$\tan z \longrightarrow \arctan z$

Calculus: derivs, int., limit

$f : \mathbb{R} \to \mathbb{R}$

$$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

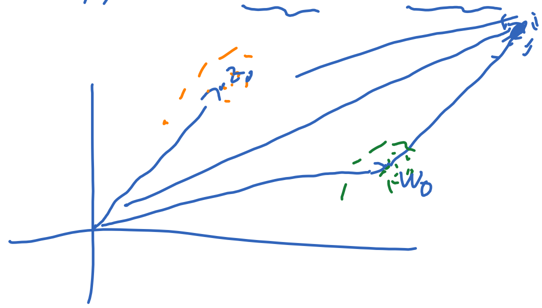
$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{k=0}^{n-1} f(x_k)(x_{k+1} - x_k)$$



Limit: (z_n) ... sequence in $\mathbb{C}$.

$\lim_{n\to\infty} z_n = z_0$ if $\underbrace{|z_n - z_0|}_{\text{real}} \to 0$ as $n \to \infty$.

$$\lim (z_n + w_n) = \underbrace{\lim z_n}_{} + \underbrace{\lim w_n}_{}$$



$$\lim (z_n w_n) = (\lim z_n)(\lim w_n)$$

$$\lim \frac{z_n}{w_n} = \frac{\lim z_n}{\lim w_n} \longrightarrow \text{assuming } \lim w_n \neq 0$$

$$\lim z_n = z_0 \overset{def}{\Longleftrightarrow} |z_n - z_0| \longrightarrow 0$$

$$\begin{cases} z_n = x_n + i y_n \\ z_0 = x_0 + i y_0 \end{cases}$$

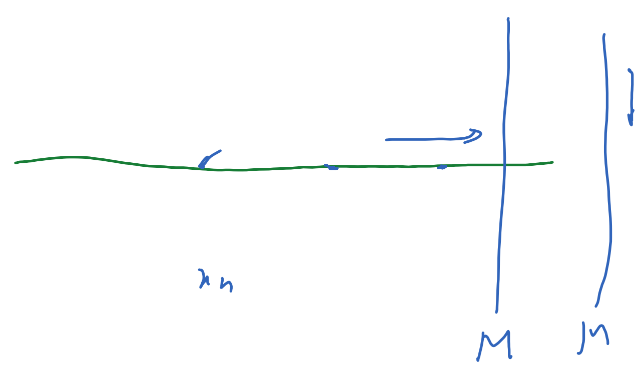
$$|z_n - z_0| = \sqrt{(x_n - x_0)^2 + (y_n - y_0)^2} \longrightarrow 0$$

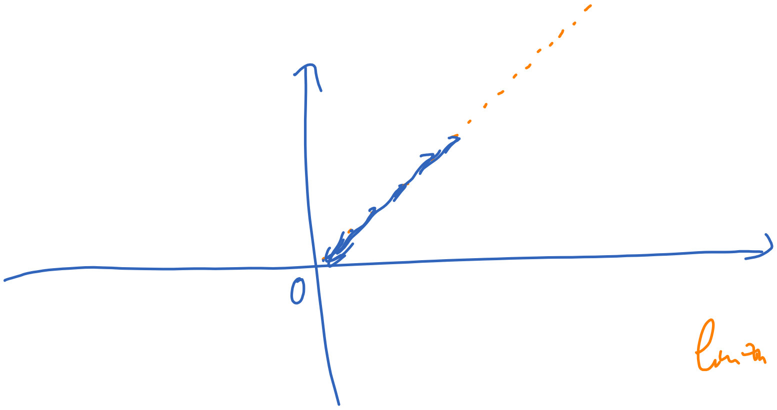
$$\lim z_n = z_0 \quad \text{iff} \quad \begin{cases} x_n \to x_0 \\ y_n \to y_0 \end{cases}$$

$\lim z_n = \infty$??

$x_n \in \mathbb{R}$

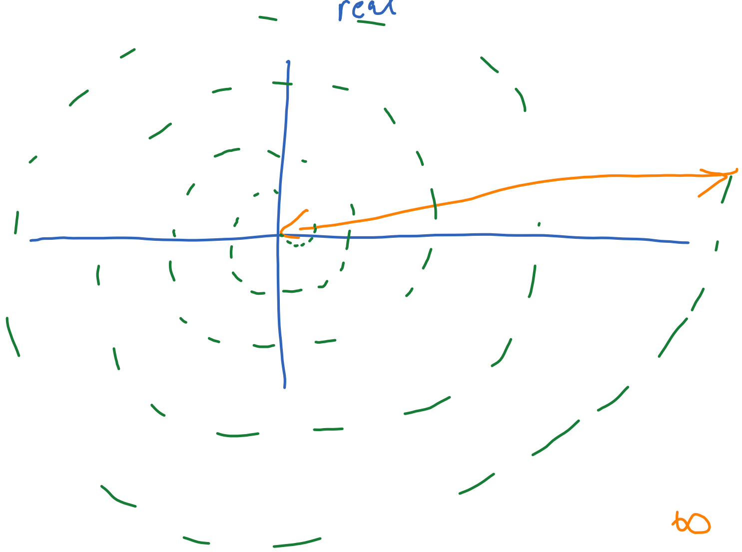
What does $\lim_{n \to \infty} x_n = \infty$ mean?





$\lim z_n = \infty$ because $|z_n| \to \infty$

$\lim z_n = \infty$ iff $\underbrace{|z_n|}_{\text{real}} \to \infty$

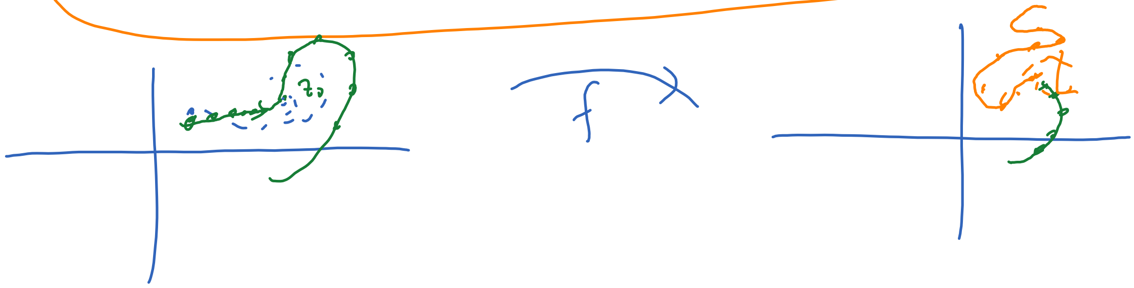


$z_n \overset{?}{\to} \infty$

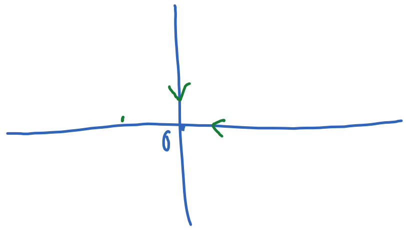
$\lim z_n = \infty$

- limit of sequence.
- limit of function:

$$\lim_{z \to z_0} f(z) = L \quad \text{if} \quad f(z_n) \to L \text{ for any sequence } z_n \to z_0.$$



$\lim_{z\to 0} \frac{\bar{z}}{z} = ?$



$z_n = \frac{1}{n}$

$\bar{z}_n = \frac{1}{n}$

$\frac{\bar{z}_n}{z_n} = 1$

$\lim \frac{\bar{z}}{z}$ DNE.

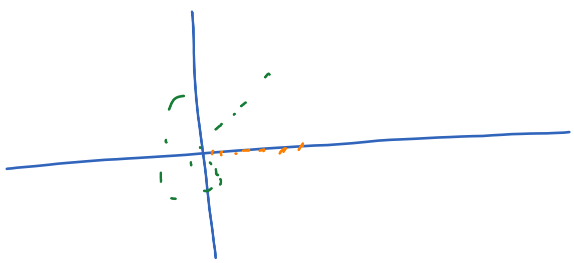
$z_n = i\frac{1}{n}$

$\bar{z}_n = -i\frac{1}{n}$

$\frac{\bar{z}_n}{z_n} = -1$

diff.

$\lim_{z\to 0} z^2$



Let z_n be a sequence with $\lim z_n = 0$.

Want to see if $z_n^2 \to 0$

$$z_n^2 = \underset{\downarrow\atop 0}{z_n} \cdot \underset{\downarrow\atop 0}{z_n} \longrightarrow 0 \cdot 0 = 0.$$

Conclusion: $\lim_{z\to 0} z^2 = 0$

$\lim_{z\to\infty} \frac{z+i}{iz+1} = ?$

$\lim_{x\to\infty} \frac{ax+b}{cx+d} = \frac{a}{c}$

$$\frac{ax+b}{cx+d} = \frac{a+\overbrace{\frac{b}{x}}^{\to 0}}{c+\underbrace{\frac{d}{x}}_{\to 0}} \to \frac{a}{c}$$

Let z_n be a sequence s.t. $\lim z_n = \infty$.

want to show $\lim_{n\to\infty} \frac{z_n + i}{iz_n + 1} = \frac{1}{i}$.

$$\frac{z_n + i}{iz_n + 1} = \frac{1 + \frac{i}{z_n}}{i + \frac{1}{z_n}} \longrightarrow \frac{1 + 0}{i + 0} = \frac{1}{i}$$

want to see if

$$\lim \frac{i}{z_n} = 0$$

$$\left| \frac{i}{z_n} - 0 \right| = \left| \frac{i}{z_n} \right| = \frac{|i|}{|z_n|} = \frac{1}{|z_n|} \longrightarrow 0$$

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