Homework 4

Due 5/1/2020

1. Find the following limits. Distinguish between the limit that is equal to ∞ and limit that does not exist. If the limit is a complex number, write your answers in either standard or polar form.

   (a) $\lim_{z\to i+1} \frac{(z-i)(2z+1)}{z+1}$

   (b) $\lim_{z\to i} \frac{z^2+1}{(z-i)(z+1)}$

   (c) $\lim_{z\to -i} \frac{z^3-i}{(z+i)(z-1)}$

   (d) $\lim_{z\to e^{i\frac{\pi}{3}}} \frac{z}{z^3+1}$

   (e) $\lim_{z\to\infty} \frac{z^2+1}{(z-2i)(z+1)}$

   (f) $\lim_{z\to\infty} \frac{z^2+1}{z+1}$

   (g) $\lim_{z\to\infty} \frac{\bar{z}}{z}$

   (h) $\lim_{z\to\infty} e^{-1/z^2}$

   (i) $\lim_{z\to\infty} \sin z$

   (j) $\lim_{z\to\infty} \frac{\sin z}{z}$

2. Determine the region of continuity of the following functions.

   (a) $f(z) = \mathrm{Log}(z^2+1)$

   (b) $f(z) = \mathrm{Log}(z^2+i)$

   (c) $f(z) = \sqrt{z-1}\sqrt{z+i}$ (principal logarithm being used)

   (d) $f(z) = (z^2+1)^i$ (principal logarithm being used)

Before doing the following problems, please take a look at the supplemental material called "**Guessing region of continuity and visualizing multivalued functions**" posted on Canvas and course website. *Make sure to include the Mathematica codes and figures you use and some brief comments.*

3. Let $f(z) = \sqrt[3]{z}$ with the principal logarithm being used.

   (a) Plot the real and imaginary part of $f(z)$.

   (b) Determine the region of continuity of f.

4. Put $g(z) = \sqrt[3]{z}$ where the regular logarithm is used. This is a multivalued function.

   (a) Plot the real part (all branches in one graph) of g.

   (b) Plot the imaginary part (all branches in one graph) of g.