Homework 2

Due 4/17/2020

1. Find all complex roots of the following cubic equation. Write them in standard form $z = a+ib$ where a and b are numerical values (round to 4 digits after decimal point).

   (a) $z^3 + 3z + 1 = 0$

   (b) $2z^3 - 6z^2 + 2z + 1 = 0$

2. Express the following complex numbers in either standard form or polar form.

   (a) $e^{e^{1+2i}}$

   (b) $e^{|2-i|}$

   (c) $\sin(-1+i)$

   (d) $\tan(i)$

3. Recall de Moivre's formula:

$$(\cos x + i \sin x)^n = \cos(nx) + i \sin(nx)$$

   for any real number x and integer number n. Apply this formula to express $\cos(5x)$ and $\sin(5x)$ in terms of $\cos x$ and $\sin x$.

4. Recall that the sine and cosine functions are defined in terms of the exponential function. Use the identity $e^{u+v} = e^u e^v$ for any $u, v \in \mathbb{C}$ to prove the following identity:

$$\sin(z+w) = \sin z \cos w + \cos z \sin w \qquad \forall z, w \in \mathbb{C}.$$

5. For $z, w \in \mathbb{C}$, show the following identities.

   (a) $\overline{z+w} = \bar{z} + \bar{w}$

   (b) $\overline{zw} = \bar{z}\bar{w}$

   (c) $|zw| = |z||w|$

   (d) $\overline{\left(\frac{z}{w}\right)} = \frac{\bar{z}}{\bar{w}}$ where $w \neq 0$

   (e) $|z^n| = |z|^n$ where n is a positive or negative integer

6. Before doing the following problem, please take a look at the supplemental material called "**Mapping properties of the exponential function**" posted on Canvas (or course website). *Make sure to include the Mathematica codes you use and some brief comments.*

   (a) Use Mathematica to plot the image of the line $y = -1$ under the function $f(z) = z^2$.

   (b) Use Mathematica to plot the image of the unit circle $x^2 + y^2 = 1$ under the function $f(z) = z^2$.

   (c) Do Part (a) and (b) for $f(z) = z^3$.

   (d) Do Part (a) and (b) for $f(z) = 1/z$.