Worksheet

12/06/2019

1. Let $V = P_2(\mathbb{R})$ be the vector space of all polynomials of degree ≤ 2 with real coefficients. Let

$$\begin{aligned} V_1 &= \{f \in V : \ f(1) = 0\}\,, \\ V_2 &= \{f \in V : \ f(2) = 0\}\,. \end{aligned}$$

   Is $V_1 + V_2$ a direct sum ?

2. Let $V = P_2(\mathbb{R})$. Define $\phi(u) = |u(1)| + |u(2)|$ for any $u \in V$. Is ϕ a norm on V ?

3. Let $V = P_2(\mathbb{R})$. Define $\phi(u) = |u(1)| + |u(2)| + |u(3)|$ for any $u \in V$. Show that ϕ is a norm on V.

4. Let $V = M_{2\times 2}(\mathbb{R})$. Let $f : V \to V$ be a linear map defined by $f(A) = A^T$. Is f diagonalizable? If it is, find a basis of V in which f is represented by a diagonal matrix.