Lecture 11 (10/18/2019)

Practice on finding coordinate vectors, null and range space: see worksheets.

Note: the section in the textbook that talks about coordinate vectors and matrix representation of linear maps is Section 8 of Chapter 1.

$f: V \to W$ linear

null(f) is a subspace of V:

$$\text{null}(f) = \{v \in V : f(v) = 0\}.$$

range(f) is a subspace of W:

$$\text{range}(f) = \{f(v) : v \in V\}.$$

The dimension of null(f) is called the nullity of f.

The dimension of range(f) is called the rank of f.

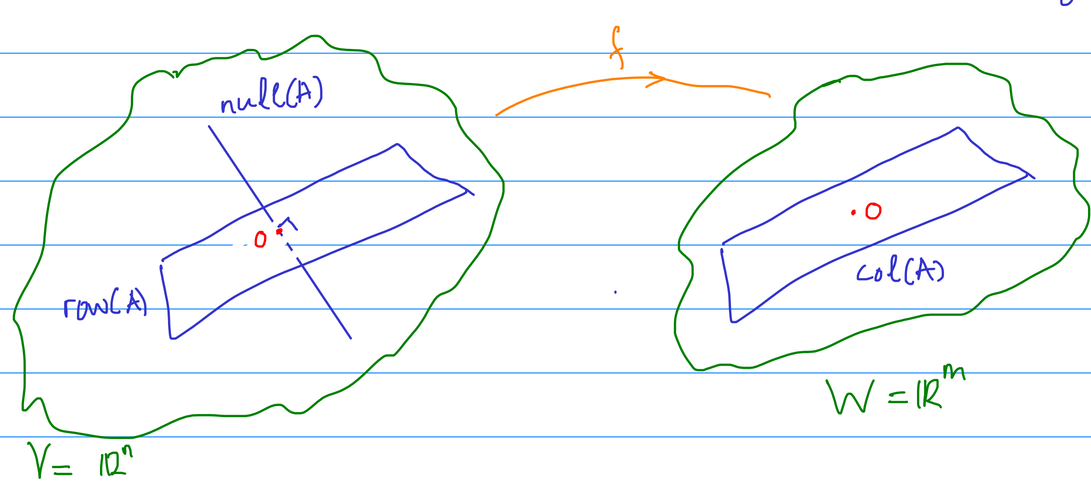
* Null-rank theorem:

Recall that if A is an $m \times n$ matrix then

$$\text{null}(A) + \text{rank}(A) = n \quad \text{(null-rank theorem)}$$

range(A) is the same as col(A), the column space of A.

The following picture summarizes the null-rank theorem in Math 341: $f: \mathbb{R}^n \to \mathbb{R}^m$ is the linear map represented by A.



- f maps everything in null(A) to $\{0\}$.
- f maps row(A) one-to-one and onto col(A). Thus,

$$\dim \text{row}(A) = \dim \text{col}(A) \quad (= \text{rank of } A)$$

- null$(A) \perp$ row(A) because

$$Av = \begin{bmatrix} — R_1 — \\ — R_2 — \\ \vdots \\ — R_m — \end{bmatrix} \begin{bmatrix} | \\ v \\ | \end{bmatrix} = \begin{bmatrix} R_1 \cdot v \\ R_2 \cdot v \\ \vdots \\ R_m \cdot v \end{bmatrix}.$$

For any $v \in$ null(A), $Av = 0$. Thus, RHS $= 0$. This implies $R_1, R_2, \ldots, R_m$ are perpendicular to v. Therefore,

$$\text{row}(A) = \text{span}\{R_1, R_2, \ldots, R_m\} \perp \text{null}(A).$$