Homework 7

Due 11/25/2019

In the following problems, make sure to write your arguments coherently in full sentences. If possible, start a sentence with words rather than a formula. Avoid using ambiguous symbols such as ?, ..., ∴. You can use the arrows to indicate row operations. In other circumstances, use words to transition your ideas, for example "This leads to", "Therefore", "We want to show", etc.

1. Let $T : V \to V$ be a linear map. Show that $\text{null}(T)$ and $\text{range}(T)$ are invariant subspaces under T.

2. Let $T : V \to V$ be a linear map. For each $n \geq 1$, denote by T^n the composition mapping $T \circ T \circ \ldots \circ T$ (n times). Show that $\text{null}(T) \subset \text{null}(T^2)$.

3. Let
$$A = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 6 & 2 & 0 & 0 \\ 6 & 2 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}.$$
Put
$$\begin{aligned} V_1 &= \{v : \ Av = (-1)v\}, \\ V_2 &= \{v : \ Av = 0v\}, \\ V_3 &= \{v : \ Av = 2v\}. \end{aligned}$$
Show that $V_1 \oplus V_2 \oplus V_3 = \mathbb{R}^4$.

4. Consider a subspace of $P_3(\mathbb{R})$
$$V_1 = \left\{ u \in P_3(\mathbb{R}) : \ u(1) = u'(1) = 0 \right\}.$$
Find a subspace V_2 of P_3 such that $V_1 \oplus V_2 = P_3$.
Hint: convert the problem in P_3 to a problem in $\mathbb{R}^4$ by using coordinates.

Do the following problem for 6 bonus points.

5. Consider two subspaces of $\mathbb{C}^3$:
$$\begin{aligned} V_1 &= \left\{ (z_1, z_2, z_3) \in \mathbb{C}^3 : \ z_1(1+i) + 2z_2 = 0 \right\}, \\ V_2 &= \left\{ (z_1, z_2, z_3) \in \mathbb{C}^3 : \ z_2 + (2-i)z_3 = 0 \right\}. \end{aligned}$$
Find a basis of $V_1 + V_2$. Is it a direct sum?