Homework 4

Due 10/25/2019

In the following problems, make sure to write your arguments coherently in full sentences. If possible, start a sentence with words rather than a formula. Avoid using ambiguous symbols such as $\rightarrow$, ?, $\ldots$, $\therefore$. Instead, use words to transition your ideas, for example "This leads to", "Therefore", "We want to show", etc.

For Problem 1, 2, 3, let V be the set of all 3×2 matrices with real coefficients such that the sum of the entries on each row is equal to 0.

1. Show that V is a vector space over $\mathbb{R}$.
2. Find a basis of V. Call it $\mathcal{B}$. What is $\dim_{\mathbb{R}} V$?
3. Find the coordinate vector (in basis $\mathcal{B}$) of the following matrix:

$$A = \begin{bmatrix} 2 & -2 \\ -3 & 3 \\ 0 & 0 \end{bmatrix}$$

For Problem 4, 5, 6, 7, let $f : M_{2\times 2}(\mathbb{R}) \rightarrow P_2(\mathbb{R})$ be a function defined as

$$f\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = a(x-1)^2 + bx$$

Here $P_2(\mathbb{R})$ denotes the vector space of all polynomials with real coefficients of degree ≤ 2.

4. Show that f is a linear map.
5. Find a matrix representation of f.
6. Find a basis of null(f). What is the nullity of f?

Do the following problem for 6 bonus points.

7. Find a basis of range(f). What is the rank of f?