Homework 3

Due 10/18/2019

In the following problems, make sure to write your arguments coherently in full sentences. Avoid using ambiguous symbols such as →, ?, …, ∴. Instead, use words to transition your ideas, for example "This leads to", "Therefore", "We want to show", etc.

Let

$$V = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \in \mathbb{R}, \ a + b + c = 0 \right\},$$

W be the set of all functions of the form $\alpha x + \beta x^2 + \gamma e^x$ where $\alpha, \beta, \gamma \in \mathbb{R}$, $\alpha + \beta + \gamma = 0$.

1. Show that V is a vector space over $\mathbb{R}$.

2. Find a basis of V. Name it B_1. What is the dimension of V ?

3. Show that W is a vector space over $\mathbb{R}$.

4. Find a basis of W. Name it B_2. What is the dimension of W ?

5. Consider a function $f : V \to W$ given by

$$f\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} \right) = ax + bx^2 + ce^x.$$

   Show that f is a linear map.

6. Find the matrix $[f]_{B_2, B_1}$.

Do the following problem for 6 bonus points.

7. Let V be the set of all continuous functions from the interval $[0, 1$ to $\mathbb{R}$. Consider a map $F : V \to \mathbb{R}$ given by,

$$F(u) = \int_0^1 x^2 u(x) dx \qquad \forall\, u \in V.$$

   Is F linear? Verify your answer.