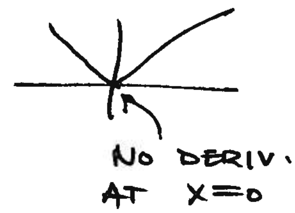
14 November 2014 **Exam [redacted] Review**

KEY

(1) State whether each statement is true or false.

(a) If f is continuous on $(-\infty, \infty)$ then f has a derivative defined on $(-\infty, \infty)$.

FALSE. EX: $f(x) = |x|$



(b) If f is continuous on $(-\infty, \infty)$ then f has an antiderivative defined on $(-\infty, \infty)$.

TRUE. BY FTC 1, $F(x) = \int_a^x f(t)\,dt$ IS AN ANTIDERIVATIVE OF f.

(c) $\int_{-1}^{1} \left(x^5 - 6x^9 + \frac{\sin x}{(1+x^4)^2} \right) dx = 0$

TRUE. $x^5 - 6x^9 + \frac{\sin x}{(1+x^4)^2}$ IS AN ODD FUNCTION.

(d) If $f(x)$ is continuous on $[a, b]$ then

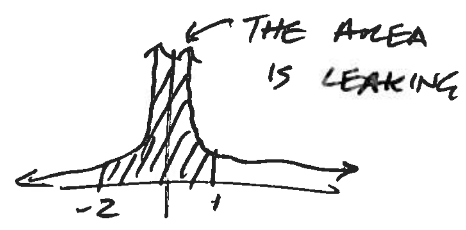
$$\frac{d}{dx} \int_a^b f(x)\,dx = f(x)$$

FALSE. $\int_a^b f(x)\,dx$ IS A NUMBER

SO $\frac{d}{dx} \int_a^b f(x)\,dx = 0$.

(e) $\int_{-2}^{1} \frac{1}{x^4}\,dx = -\frac{3}{8}$

FALSE. $f(x) = \frac{1}{x^4}$ IS NOT CONTINUOUS AT $x=0$.



(2) Evaluate the definite integral. Simplify your answer.

$$\int_{-5}^{5} 2\sqrt{25 - x^2}\, dx$$

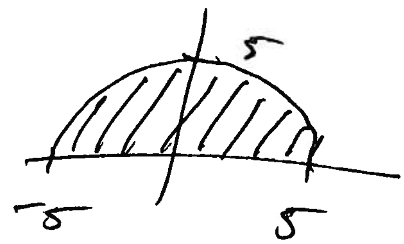
$y = \sqrt{25 - x^2}$

IS THE EQUATION OF THE TOP-HALF OF A CIRCLE OF RADIUS 5.

$= 2 \cdot \text{AREA}(\;)$

$= 2 \cdot \frac{1}{2} \pi (5)^2$

$= 25\pi$



(3) Evaluate the definite integral. Simplify your answer.

$$\int_{2}^{3} \frac{12x}{(x^2 - 1)^2}\, dx$$

$u = x^2 - 1$

$du = 2x\, dx$

$x = 2: \; u = 3$

$x = 3: \; u = 8$

$$= \int_{3}^{8} \frac{6}{u^2}\, du = \left. \frac{-6}{u} \right|_{3}^{8} = \frac{-6}{8} + \frac{6}{3}$$

$$= \frac{5}{4}$$

(4) Evaluate the indefinite integral.

$$\int \frac{(5-\ln x)^4}{x}\,dx$$

$u = 5 - \ln x$

$du = \frac{dx}{x}$

$$= \int u^4\,du = \frac{u^5}{5} + C$$

$$= \frac{(5-\ln x)^5}{5} + C$$

(5) Evaluate the indefinite integral.

$$\frac{1}{2}\int 2x^3\sqrt{x^2+3}\,dx$$

$u = x^2+3 \rightsquigarrow x^2 = u-3$

$du = 2x\,dx$

$$= \frac{1}{2}\int (u-3)\sqrt{u}\,du$$

$$= \frac{1}{2}\int \left(u^{3/2} - 3u^{1/2}\right) du$$

$$= \frac{1}{2}\left[\frac{2}{5}u^{5/2} - 2u^{3/2}\right] + C$$

$$= \frac{1}{5}(x^2+3)^{5/2} - (x^2+3)^{3/2} + C$$

(6) Evaluate the indefinite integral.

$$\int \tan x \sec^5 x \, dx$$

$u = \sec x$

$du = \sec x \tan x \, dx$

$= \int \tan x \sec x \sec^4 x \, dx$

$= \int u^4 \, du$

$= \frac{u^5}{5} + C = \frac{\sec^5 x}{5} + C$

(7) A particle moves along the x-axis with position $x(t)$, velocity $v(t)$, and acceleration $a(t) = \sin(2\pi t)$. Given that $x(1/2) = 7$ and $v(1/4) = 0$, find the particle's position at time $t = 5$.

SORRY ABOUT THE MESS

$\downarrow$

$= \sin(2\pi t)$

$V = -\frac{1}{2\pi}\cos(2\pi t) + C$

$0 = -\frac{1}{2\pi}\cos\left(2\pi \cdot \frac{1}{4}\right) + C$ $\qquad \cos\left(\frac{\pi}{2}\right) = 0$

$0 = C$

$v(t) = -\frac{1}{2\pi}\cos(2\pi t)$

$s(t) = -\frac{1}{(2\pi)^2}\sin(2\pi t) + D$ $\qquad \sin(\pi) = 0$

$7 = -\frac{1}{(2\pi)^2}\sin(\pi) + D$

$D = 7$

$s(t) = -\frac{1}{(2\pi)^2}\sin(2\pi t) + 7$

$s(5) = -\frac{1}{(2\pi)^2}\sin(10\pi) + 7$

$= \boxed{7}$

(8) Evaluate the following sum/limit (look in your notes for the summation formulas).

$$\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{10k}{n^2} + \frac{3}{n+2} \right)$$

$$= \lim_{n\to\infty} \left(\frac{10}{n^2} \sum_{k=1}^{n} k + \frac{3}{n+2} \sum_{k=1}^{n} 1 \right)$$

$$= \lim_{n\to\infty} \left(\frac{10}{n^2} \cdot \frac{n(n+1)}{2} + \frac{3}{n+2} \cdot n \right)$$

$$= \quad 5 + 3 = \boxed{8}$$

(9) Find an equation of the line tangent to the graph of $f(x) = \int_x^{x^2} e^{t^2}\,dt$ at the point $x = 0$.

$$f(x) = -\int_0^x e^{t^2} dt + \int_0^{x^2} e^{t^2} dt$$

$$f'(x) = -e^{x^2} + e^{x^4} \cdot 2x$$

TAN. LINE: $y - y_1 = m(x - x_1)$

$y_1 \leftarrow f(0)$, $m \leftarrow f'(0)$, $x_1 \leftarrow 0$

$$f'(0) = -e^0 + e^0 \cdot 0 = -1$$

$$f(0) = \int_0^0 e^{t^2} dt = 0$$

$$\boxed{y - 0 = -1(x - 0)}$$

(10) Suppose that $f(1) = 10$ and that $f'(x) \geq 2$ for $1 \leq x \leq 4$. Use the mean value theorem to determine the smallest value that $f(4)$ can be.

NOT THE MVT FOR INTEGRALS

MVT:

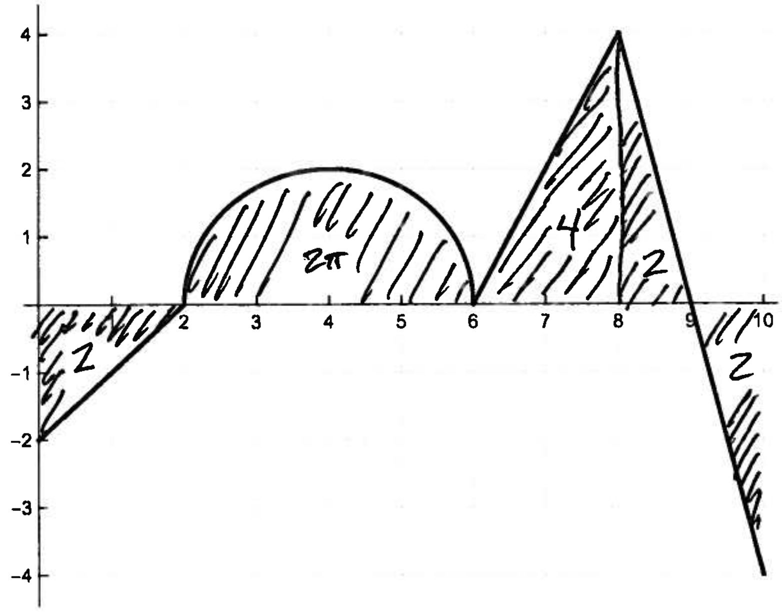
THERE IS A c IN $(1,4)$ SUCH THAT

$$f'(c) = \frac{f(4) - f(1)}{4 - 1}$$

$$\Rightarrow \frac{f(4) - 10}{3} = f'(c) \geq 2$$

$$\Rightarrow f(4) \geq \underline{\underline{16}}$$

(11) The graph below shows the velocity $v(t)$ of a particle. You may assume that any curve that looks like a circle is, in fact, a circle.



(a) What is the displacement of the particle from $0 \le t \le 10$?

$$\text{DISP} = \int_0^{10} v(t)\,dt = -2 + 2\pi + 4 + 2 - 2$$

$$= \boxed{2 + 2\pi}$$

(b) What is the total distance that the particle traveled from $0 \le t \le 10$?

$$\text{TOT DIST} = \int_0^{10} |v(t)|\,dt = 2 + 2\pi + 4 + 2 + 2$$

$$= \boxed{10 + 2\pi}$$

(c) What is the average value of the velocity $v(t)$ for $6 \le t \le 9$?

$$\text{Av} = \frac{1}{9-6}\int_6^9 v(t)\,dt = \frac{1}{3}(4+2) = \boxed{2}$$

(12) Suppose that f is an odd function and g is an even function, and you are given the values of the following integrals:

$$\int_3^5 f(x)\,dx = 7$$
$$\int_0^5 g(x)\,dx = -3$$
$$\int_{-1}^0 g(x)\,dx = -1$$

Evaluate the following definite integrals.

(a) $\int_4^4 [f(x)]^2\,dx$ $= 0$

(b) $\int_{-3}^5 2f(x)\,dx$ $= 2\int_{-3}^{3} f(x)\,dx + 2\int_3^5 f(x)\,dx$

$= \underset{\text{(ODD)}}{0} + 2\cdot 7 = \boxed{14}$

(c) $\int_{-5}^5 (12f(x) - 4g(x))\,dx$ $= 12\int_{-5}^{5} f(x)\,dx - 4\int_{-5}^{5} g(x)\,dx$

$= \underset{\text{(ODD)}}{0} - \underset{\text{(EVEN)}}{4\cdot 2\cdot(-3)} = \boxed{24}$

(d) $\frac{1}{2}\int_0^1 2x\, g(x^2)\,dx$

$u = x^2$

$du = 2x\,dx$

$= \frac{1}{2}\int_0^1 g(u)\,du \underset{\text{(EVEN)}}{=} \frac{1}{2}\int_{-1}^0 g(u)\,du = \boxed{\frac{-1}{2}}$