12 November 2014 **Miscellaneous Integral Stuff**

(1) Find the absolute max/min of the function $f(x) = xe^{-x}$ on $[0, 2]$. Use this information to estimate

$$\int_0^2 x\, e^{-x}\, dx.$$

(2) Find the absolute max/min of the function $f(x) = x - 2\sin x$ on $[\pi, 2\pi]$. Use this information to estimate

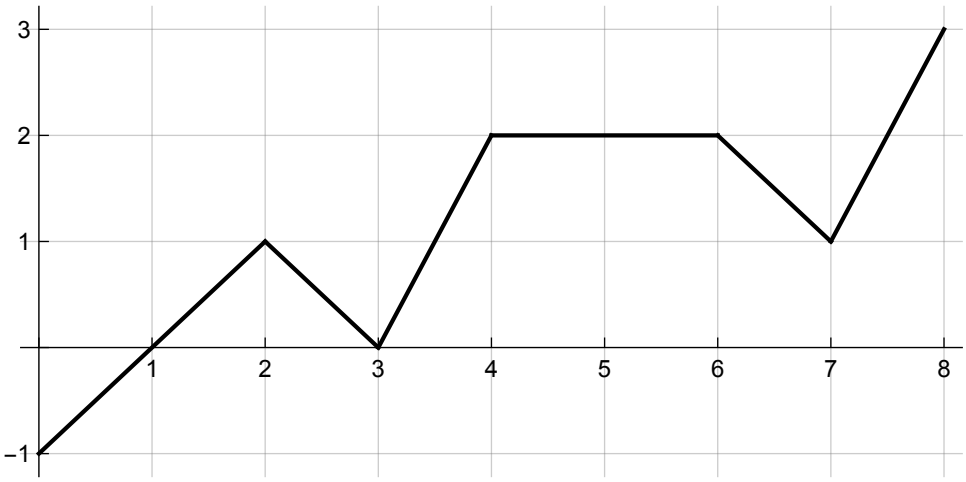
$$\int_\pi^{2\pi} x - 2\sin x\, dx.$$

You may find it useful to know that $5\pi/3 + \sqrt{3} \approx 7$.

(3) Water flows from the bottom of a storage tank at a rate of $r(t) = 200 - 4t$ liters per minute, where $0 \leq t \leq 50$. Find the amount of water that flows from the tank during the first 10 minutes.

(4) If the velocity of a particle moving along a line is given by $v(t) = t^2 - 2t - 8$, find the displacement and total distance travelled between $t = 1$ and $t = 8$.

(5) The graph of a function $f(x)$ is shown below. Find the average value of f on $[0, 8]$.



(6) Find the average value of $f(x) = 2\sin x - \sin(2x)$ on the interval $[0, \pi]$.

(7) Suppose that $f(x) = (x-3)^2$. Find a number c in $[2,5]$ such that

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx.$$

Sketch the graph of $f(x)$ and a rectangle whose area is the same as the area under the graph of f.