11 November 2014 **The Fundamental Theorem of Calculus Part 1**

(1) Find the derivative of each of the following functions.

(a) $f(x) = \displaystyle\int_x^{\pi} \sqrt{1+\sec t}\, dt$

(b) $g(x) = \displaystyle\int_{-e}^{\sqrt{x}} \frac{5z^2}{z^4+1}\, dz$

(c) $h(x) = \displaystyle\int_{e^{-x}}^{\sin x} \ln(1+2v^2)\, dv$

(2) Suppose $w(x) = \displaystyle\int_{10}^{x^2} (t-4)(t+1)^6\, dt$. Determine all the intervals on which $w(x)$ is increasing.

(3) Recall that the Fresnel $S(x)$ function is defined by $S(x) := \int_0^x \sin\left(\frac{\pi t^2}{2}\right)\, dt$. Make a rough sketch of the graph of $S(x)$ using $S'(x)$ and $S''(x)$ (like we have done many times before).

(4) Evaluate $\lim_{x\to 0} \frac{1}{x^3} \int_0^x \sin\left(\frac{\pi t^2}{2}\right)\, dt$. Hint: L'Hospital's rule.

(5) Find a function f and a number a such that $6 + \int_a^x \frac{f(t)}{t^2}\, dt = 2\sqrt{x}$.