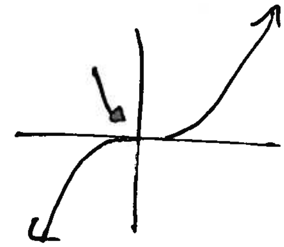
(1) Is the given statement **true** or **false**?

(a) If $f'(c) = 0$ then f has a local minimum or maximum at c.

FALSE

EX: $y = x^3$



(b) $\lim_{x\to\infty} \frac{e^{-x} \;\to 0}{\ln(1/x) \;\to -\infty} = 0$

TRUE

(c) If $\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = 0$ then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}.$$

FALSE

L'HOSPITAL'S RULE IS

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

(d) $\lim_{x\to 0} \frac{x^{100} - 1}{x - 1} = 100.$

FALSE. THE LIMIT IS 1.

(2) Find $g'(x)$ given that $g(x) = 5^x + x^5 + \sin 5x + \sin^{-1} 5x$

$$g'(x) = 5^x \ln 5 + 5x^4 + 5\sin 5x + \frac{1 \cdot 5}{\sqrt{1-(5x)^2}}$$

(3) Find $\dfrac{dz}{dy}$ given that $z = \tan^2(\sin y + \cos 2y)$ $= [\tan(\sin y + \cos(2y))]^2$

$$\frac{dz}{dy} = 2\tan(\sin y + \cos 2y)(\cos y - 2\sin 2y)$$

(4) Find $f'(t)$ given that $f(t) = \dfrac{\ln t}{(t+7)^3}$

$$f'(t) = \frac{(t+7)^3 \cdot \frac{1}{t} - \ln t \cdot 3(t+7)^2}{(t+7)^6}$$

(5) Use linear approximation or differentials to approximate $\sqrt[3]{9}$.

SINCE $\sqrt[3]{8} = 2$ IS EASY TO EVALUATE, WE SET $x_1 = 8$. THEN $y_1 = 2$.

$$y = \sqrt[3]{x} \longrightarrow \frac{dy}{dx} = \frac{1}{3}x^{-2/3} \longrightarrow \left.\frac{dy}{dx}\right|_{x_1=8} = \frac{1}{12}$$

TAN. LINE : $y - 2 = \frac{1}{12}(x-8) \rightsquigarrow y = \frac{1}{12}(x-8) + 2$

SO $\sqrt[3]{9} \approx \frac{1}{12}(9-8) + 2 = \boxed{2 + \frac{1}{12}}$

(6) Find the equation of the line tangent to the curve

$$y^2 - xy + x^3 = 4x - 7y$$

at the point $(2, 0)$.

$$2y\frac{dy}{dx} - \left(x\frac{dy}{dx} + y\right) + 3x^2 = 4 - 7\frac{dy}{dx}$$

$$2y\frac{dy}{dx} - x\frac{dy}{dx} + 7\frac{dy}{dx} = 4 - 3x^2 + y$$

$$\frac{dy}{dx} = \frac{4 - 3x^2 + y}{2y - x + 7}$$

$$\left.\frac{dy}{dx}\right|_{(2,0)} = \frac{4 - 3(2)^2 + 0}{2(0) - 2 + 7} = \frac{-8}{5}$$

$$\boxed{y - 0 = -\frac{8}{5}(x - 2)}$$

(7) A function $f(x)$ has domain $[0, \infty)$ and has the following derivative.

$$f'(x) = x(x^2 - 4)(x - 7)(e^{3x} + \sqrt{x})$$

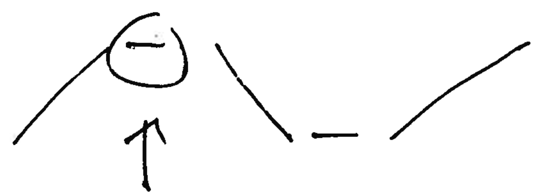
ALWAYS POSITIVE FOR $x > 0$

Determine the x value of every local maximum of $f(x)$.

CRITICAL PTS $x = \cancel{-2}, 0, 2, 7$

OUTSIDE DOMAIN.

x	0	$(0,2)$	2	$(2,7)$	7	$(7, \infty)$
$f'(x)$	0	+	0	−	0	+



LOCAL MAX AT $\boxed{x = 2}$

(8) Boyle's Law states that when a sample of gas is compressed at a constant temperature, the pressure P and volume V satisfy the equation

$$PV = k$$

where k is a **constant**. Suppose that at a certain instant, the volume is 60, the pressure is 15, and the pressure is decreasing at a rate of 2 (we won't worry about units here). At what rate is the volume increasing at this instant?

GIVEN: $\frac{dP}{dt} = -2$

WANT: $\frac{dV}{dt} = ?$

$$P \frac{dV}{dt} + \frac{dP}{dt} V = 0$$

$$(15) \frac{dV}{dt} + (-2)(60) = 0$$

$$\frac{dV}{dt} = \frac{2 \cdot 60}{15} = \boxed{8}$$

$f, g > 0 \quad f', g' < 0 \quad f'', g'' > 0$

(9) Suppose f and g are positive, decreasing, concave up functions on $(-\infty, \infty)$. What is the concavity of the function fg? Justify your answer.

LET $h = fg$ NEED h'' FOR CONCAVITY.

$$h' = fg' + f'g$$

$$h'' = fg'' + f'g' + f'g' + f''g$$

$$= \underbrace{fg''}_{>0} + \underbrace{2f'g'}_{>0 \text{ SINCE } f', g' < 0} + \underbrace{f''g}_{>0}$$

SO $h'' > 0 \Longrightarrow h$ IS <u>CONCAVE UP</u>

(10) If a resistor of R ohms is connected across a battery of 2 volts with internal resistance 100 ohms, then the power (in watts) in the external resistor is

$$P = \frac{4R}{(R+100)^2}.$$

What is the maximum value of the power?

$$\frac{dP}{dR} = \frac{(R+100)^2 \cdot 4 - 4R \cdot 2(R+100)}{(R+100)^4}$$

WANT $\frac{dP}{dR} = 0 \rightsquigarrow 4(R+100)^2 - 8R(R+100) = 0$

$\Longrightarrow 4(R^2 + 200R + 10000) - 8R^2 - 800R = 0$

$\Longrightarrow -4R^2 + 40000 = 0$

$\Longrightarrow R^2 = 10000$

$\Longrightarrow \underline{R = 100}$ (THROW OUT $R = -100$)

POWER IS MAX AT $R = 100$

$\Longrightarrow P = \frac{400}{(200)^2} = \frac{\cancel{400}^{\,1}}{\cancel{40000}_{\,100}}$

$= \frac{1}{100}$ WATTS

MAX OR MIN?

$+ \quad 100 \quad -$

(11) Evaluate the following limits.

(a) $\lim_{x\to 0^-} \frac{\sin x}{\ln(1-x)}$ $\quad \sin x \to 0, \quad \ln(1-x) \to 0$

$$\overset{L'H}{=} \lim_{x\to 0^-} \frac{\cos x}{\frac{1}{1-x}(-1)} = \frac{1}{-1} = \boxed{-1}$$

(b) $\lim_{x\to\infty} \frac{\tan^{-1} x}{e^{-x}+x}$ $\quad \tan^{-1} x \to \pi/2, \quad e^{-x}+x \to \infty$ $\longrightarrow \boxed{0}$

(c) $\lim_{x\to\infty} \left(1-\frac{3}{x}\right)^{2x}$

LET $\quad y = \left(1-\frac{3}{x}\right)^{2x}$

$$\ln y = 2x \ln\left(1-\frac{3}{x}\right)$$

$$\lim_{x\to\infty} 2x \ln\left(1-\frac{3}{x}\right) = \lim_{x\to\infty} \frac{\ln\left(1-\frac{3}{x}\right)}{\frac{1}{2x}} \quad \begin{matrix}\to 0\\ \to 0\end{matrix}$$

$$\overset{L'H}{=} \lim_{x\to\infty} \frac{\frac{1}{1-\frac{3}{x}}\left(+\frac{3}{x^2}\right)}{\left(\frac{-1}{2x^2}\right)} = \lim_{x\to\infty} \frac{-6}{1-\frac{3}{x}} = \boxed{-6}$$

NOT THE FINAL ANSWER!

$$\ln y = -6$$

$$y = \boxed{e^{-6}}$$