19 September 2014 **The Derivative as a function**

(1) Suppose that $f(x) = x^3$.
   (a) Using the limit definition of the derivative, find $f'(x)$.

   (b) Sketch $y = f(x)$ and $y = f'(x)$ on the same graph. Does your answer from part (a) make sense graphically?

(2) Suppose that $f(x) = \sqrt{x}$.
   (a) Using the limit definition of the derivative, find $f'(x)$.

   (b) What is the domain of $f(x)$? What is the domain of $f'(x)$? Does this make sense?

(3) Make a careful sketch of the graph of $f(x) = \cos(x)$. On the same graph, make a sketch of $f'(x)$ (hint: where is the derivative equal to zero?). Can you guess a formula for $\dfrac{d}{dx}\cos x$ from your graph?

(4) (a) Sketch the graph of $f(x) = x|x|$.

(b) For what values of x is f differentiable?

(c) Find a formula for $f'(x)$.

(5) Suppose f is a function that satisfies the equation

$$f(x+y) = f(x) + f(y) + x^2y + xy^2$$

for all real numbers x and y. Suppose also that

$$\lim_{x\to 0} \frac{f(x)}{x} = 1$$

(a) Find $f(0)$

(b) Find $f'(0)$

(c) Find $f'(x)$