*5 September 2014*   **Inverse functions**

(1) Find the exact value of each expression.

(a) $\log_5(\frac{1}{125})$

(b) $\log_{10}(\sqrt{10})$

(c) $\log_3 100 - \log_3 18 - \log_3 50$

(d) $\log_2(\log_2 2^{2^8})$

(2) In the theory of relativity, the mass of a particle with speed $v$ is
$$m(v) = \frac{m_0}{\sqrt{1 - v^2/c^2}},$$
where $m_0$ is the rest mass of the particle (a constant) and $c$ is the speed of light in a vacuum (also a constant). Find the inverse function $v(m)$ (that is, solve for $v$ in terms of $m$) and explain its meaning.

(3) Express the following quantity as a single logarithm.
$$\tfrac{1}{3}\log_5(x+2)^3 + \tfrac{1}{2}\left[\log_5 x - \log_5(x^2 + 3x + 2)^2\right]$$

(4) A bacterial culture is known to triple every two hours. Suppose there are initially 80 bacteria.

    (a) How many bacteria are there after 6 hours?

    (b) How many bacteria are there after $t$ hours?

    (c) When will there be $80\,000$ bacteria? Give an exact answer in terms of logs (no approximations).

(5) In this exercise we will show that $\cos(\sin^{-1} x) = \sqrt{1 - x^2}$.

    (a) Think of $\sin^{-1} x$ as "the angle $\theta$ whose sine is $x$". Draw a triangle representing this relationship (that is, a triangle that shows that $\sin\theta = \frac{x}{1}$). Make sure to fill in the length of the missing side.

    (b) What is $\cos\theta$?

    (c) Show that $\cos(\sin^{-1} x) = \sqrt{1 - x^2}$.

(6) Solve each equation for $x$.

    (a) $\log_3 x + \log_3(x - 1) = 1$

    (b) $9^x - 3^{x+1} + 2 = 0$      Hint: let $z = 3^x$.