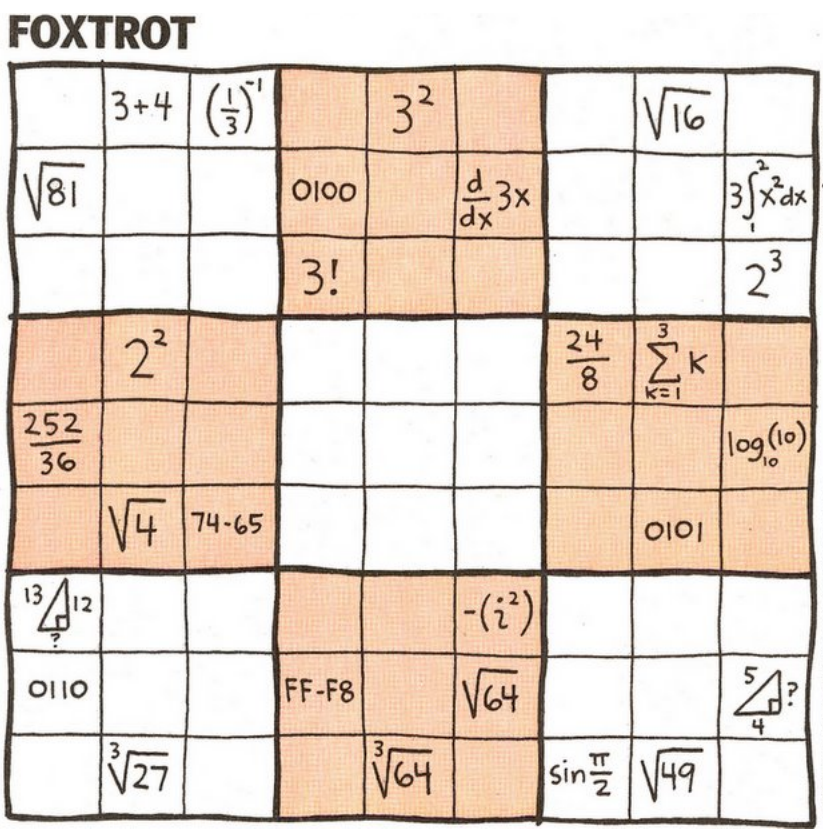
MATH 220 U1 **Exam 2** **Fall 2014**

Name ____

- Do not open this exam packet until I say *start*.
- Turn off all electronic devices and and put away all items except for a pen/pencil and an eraser.
- Remove hats and sunglasses.
- If you have a question, raise your hand and I will come to you. When you stand up, you are done with your exam.
- Quit working and close this packet when I say *stop*.
- Good luck! (If you get bored, you can try your hand at the sudoku puzzle below. . .)



POSSIBLE	[1] 12	[2] 10	[3] 18	[4] 10	[5] 20	[6] 15	[7] 10	[8] 10	[T] 100*
SCORE									

*It is possible to score a total of 105 points on this exam, but your score will be out of 100.

1. (3 points each) Find the following derivatives. No partial credit will be given for this problem.

   (a) $\frac{d}{dx} \arcsin x =$

   (b) $\frac{d}{dx} \operatorname{arcsec} x =$

   (c) $\frac{d}{dx} \log_7 x =$

   (d) $\frac{d}{dx} 19^x =$

2. (10 points) Evaluate $\frac{dy}{dx}$ at the point $(0, 1)$ given that

$$x^2 - 3xy + y^2 = 1$$

$$\boxed{\left.\frac{dy}{dx}\right|_{(0,1)} =}$$

3. (6 points each) Find the following derivatives. Please put a $\boxed{\text{box}}$ around your answer.
As a general **hint**, it is sometimes useful to simplify a function before taking its derivative.

(a) $\dfrac{d}{dx} \ln\left(\dfrac{\sqrt{x^2+1}}{(x-5)^4}\right)$

(b) $\dfrac{d}{dy}\, e^{\cos(1/y)}$

(c) $\dfrac{d}{dz}\left[(2z-\sqrt{z})(2z+\sqrt{z})\right]$

4. (10 points) A ladder 5 ft long rests against a vertical wall. The bottom of the ladder is being pushed toward the wall at a rate of 1 ft/sec. How fast is the top of the ladder moving up the wall when the bottom of the ladder is 3 ft from the wall? Please put a $\boxed{\text{box}}$ around your answer.

5. (20 points) Suppose that

$$h(x) = x^3(x-4),$$
$$h'(x) = 4x^2(x-3),$$
$$h''(x) = 12x(x-2).$$

Fill in the blanks. It might help you to sketch the graph of $h(x)$ in the space provided, but you will not be graded on your sketch. If an answer does not exist, write DNE.

(a) h is **increasing** on the interval(s)

(b) h is **decreasing** on the interval(s)

(c) h has **local maximum(s)** at $x =$

(d) h has **local minimum(s)** at $x =$

(e) h has **global maximum(s)** at $x =$

(f) h has **global minimum(s)** at $x =$

(g) h is **concave up** on the interval(s)

(h) h is **concave down** on the interval(s)

(i) h has **inflection point(s)** at $x =$

6. (5 points each) Circle the correct limit. You do not need to show any work. No partial credit will be given for this problem.

(a) $\displaystyle\lim_{x\to\infty} \frac{\sqrt{2}\,x^4}{e^{x/4}}$

$$-\infty \qquad -\sqrt{2} \qquad -1 \qquad -\frac{1}{4} \qquad 0 \qquad \frac{1}{4} \qquad 1 \qquad \sqrt{2} \qquad \infty$$

(b) $\displaystyle\lim_{x\to\infty} x^{(1/\ln(x^2))}$

$$0 \qquad \frac{1}{2} \qquad 1 \qquad \sqrt{2} \qquad \sqrt{e} \qquad e \qquad e^2 \qquad \infty$$

(c) $\displaystyle\lim_{x\to 0} \frac{1-x-e^{-x}}{x^2}$

$$-\infty \qquad -e \qquad -1 \qquad -\frac{1}{2} \qquad 0 \qquad \frac{1}{2} \qquad 1 \qquad e \qquad \infty$$

7. (10 points) $\sqrt{50}$ is slightly larger than 7, say

$$\sqrt{50} = 7 + t.$$

Use linear approximation or differentials to estimate t (your answer should be a fraction).

$t =$

8. (10 points) In this problem, we will use calculus to prove the trig identity

$$\text{if } x > 0 \text{ then} \qquad \arctan(x) + \arctan\left(\frac{1}{x}\right) = \frac{\pi}{2}. \tag{$*$}$$

(a) Let $f(x) = \arctan x + \arctan(1/x)$. Prove that $f'(x) = 0$ when $x > 0$.
(Make sure to show sufficient work.)

(b) What is $f(1)$?

(c) Using parts (a) and (b), explain why the trig identity $(*)$ is true.