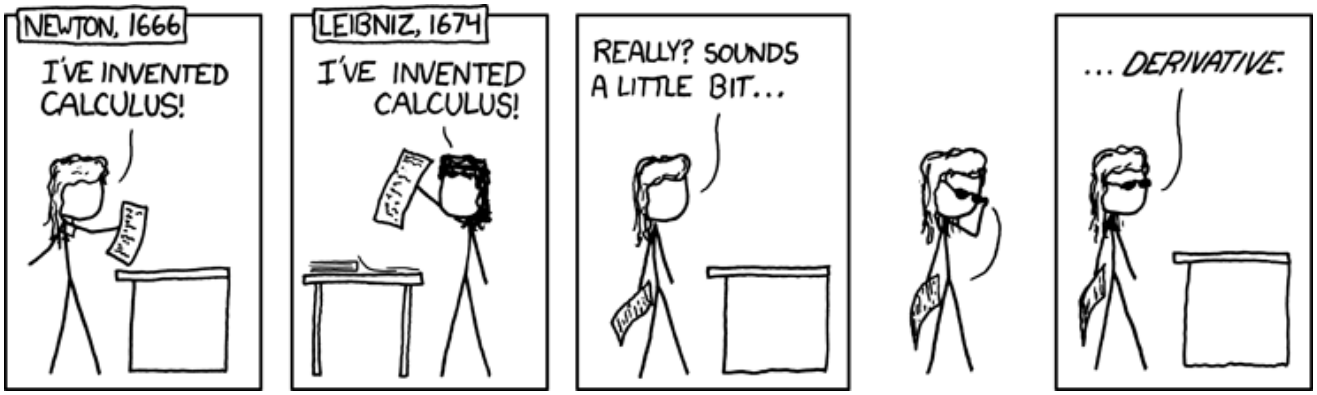
MATH 220 U1 **Exam 1** **Fall 2014**

Name _____

- Do not open this exam packet until I say *start*.
- Turn off all electronic devices and and put away all items except for a pen/pencil and an eraser.
- Remove hats and sunglasses.
- If you have a question, raise your hand and I will come to you. When you stand up, you are done with your exam.
- Quit working and close this packet when I say *stop*.
- Good luck!



SCORE	POSSIBLE
[1]	15
[2]	15
[3]	15
[4]	36
[5]	9
[6]	6
[7]	9
[T]	100*

*It is possible to score a total of 105 points on this exam, but your score will be out of 100.

1. (a) (5 points) State the definition of the derivative of $f(x)$ at the point $x = a$ (we have learned two definitions; you may state either one).

(b) (10 points) Let $f(x) = 4x - 3x^2$.
Use the definition of the derivative as a limit to prove that $f'(x) = 4 - 6x$.
Show each step in your calculation and be sure to use proper terminology at each step.

2. Recall that a function is continuous at $x = a$ if the following three conditions are satisfied:

I) $f(a)$ is defined,
II) the limit as x approaches a of $f(x)$ exists, and
III) the limit as x approaches a of $f(x)$ equals $f(a)$.

(a) (5 points) Sketch the graph of a function that satisfies I and II, but not III for $a = 2$.

(b) (10 points) Use the Intermediate Value Theorem to show that the equation

$$x^4 + x - 3 = 0$$

has a solution in $(1, 2)$.

3. (15 points) Find all **vertical and horizontal** asymptotes of the function

$$f(x) = \frac{2x^2 + x - 1}{x^2 - x - 2}.$$

Make sure to show sufficient work and write equations of lines (e.g. $x = 10$), not just numbers.

4. (6 points each) Evaluate the following limits without the use of derivatives. For infinite limits, you must state whether the limit is approaching ∞ or $-\infty$. An answer of "does not exist" is not sufficient.

(a) $\lim_{x \to 1^+} \dfrac{x^2 - 9}{x^2 + 2x - 3}$

(b) $\lim_{x \to 4^+} \dfrac{4 - x}{|4 - x|}$

(c) $\lim_{x \to -1} \dfrac{\sqrt{3 + x} - \sqrt{3}}{x}$

(d) $\lim_{x \to \pi/2^+} \dfrac{\sin(2x)}{\sin(x)}$

(e) $\lim_{x \to \infty} \arctan\left(\dfrac{x^3 - x}{x^2 - 6x + 5}\right)$

(f) $\lim_{x \to 0^+} \log_a x$ (assuming $a > 1$)

5. (3 points each) For each of the following statements, circle **True** if the statement is **always true**; otherwise circle **False**.

(a) $\lim_{x\to 1} \dfrac{x^2+8x-9}{x^2+7x-6} = \dfrac{\lim_{x\to 1} x^2+8x-9}{\lim_{x\to 1} x^2+7x-6}$

True **False**

(b) If $4x-9 \le f(x) \le x^2-4x+7$ for all $x \ge 0$ then $\lim_{x\to 4} f(x) = 7$.

True **False**

(c) If f has domain $[0,\infty)$ and f has no horizontal asymptote, then

$$\lim_{x\to\infty} f(x) = \infty \text{ or } \lim_{x\to\infty} f(x) = -\infty.$$

True **False**

6. (6 points) Suppose that $f'(3) = 7$. Circle the **three statements** below that must be true.

(a) f is an even function

(b) f is an odd function

(c) f is a one-to-one function

(d) f is differentiable at 7

(e) f is differentiable at 3

(f) f is differentiable at 0

(g) f is continuous at 7

(h) f is continuous at 3

(i) f is continuous at 0

(j) $\lim_{x\to 7} (f(x) - f(7)) = 0$

(k) $\lim_{x\to 3} (f(x) - f(3)) = 0$

(l) $\lim_{x\to 0} (f(x) - f(0)) = 0$

7. (9 points) Suppose that $\lim_{x\to a}[f(x)+g(x)]=2$ and $\lim_{x\to a}[f(x)-g(x)]=1$. Evaluate

$$\lim_{x\to a}[f(x)g(x)]$$