MATH 132 HOMEWORK 4

- (4.4) 1, 13, 15, 16, 18
  *You do not need to provide justifications for #1
  *Feel free to use the methods of later sections to compute the integrals in Section 4.4

- (4.5) 3aef, 6, 15
  *For #15: The first sentence of the hint is useful. You may want to ignore the second sentence and instead show using the generalized Cauchy Integral Formula that

$$G(z) = \begin{cases} f(z)/z & \text{if } z \neq 0, \\ f'(0) & \text{if } z = 0. \end{cases}$$

- (4.6) 4, 8

- (5.1) 11, 12

- (5.2) 5abcg, 11
  *The wording of #5 is confusing; it should be read as "Find the Taylor series and state the convergence properties of the Taylor series for the following."

- (After you have completed #15 in (4.5)) In the two parts below we will prove Schwarz's Lemma: If f is analytic in the unit disk $|z| < 1$ and satisfies the conditions

$$f(0) = 0 \quad \text{and} \quad |f(z)| \leq 1 \text{ for all } |z| < 1,$$

  then $|f(z)| \leq |z|$ for all $|z| \leq 1$.
  a) Let $F(z) = f(z)/z$ for $z \neq 0$, and $F(0) = f'(0)$. You already showed in #15 of (4.5) that $F(z)$ is analytic in $|z| < 1$. Fix ζ in the open unit disk and let r be a real number satisfying $|\zeta| < r < 1$. Use the maximum modulus principle to show that

$$|F(\zeta)| \leq \max_{|z|=r} \frac{|f(z)|}{r} \leq \frac{1}{r}.$$

  b) Letting $r \to 1^-$ in the inequality above, deduce that $|F(\zeta)| \leq 1$, and thus $|f(\zeta)| \leq |\zeta|$ for all $|\zeta| < 1$.

Extra Practice (do not turn in):
(4.4) 3, 9, 10, 11, 14, 17, 19 (4.5) 2, 3bcd, 4, 7, 9 (4.6) 7, 10, 14, 16
(5.1) 1, 2, 7, 14 (5.2) 1, 2, 4, 5def