MATH 132 HOMEWORK 1

- (1.1) 21, 22, 23, 24, 30
- (1.2) 7ehij, 16
- (1.3) 11
- (1.4) 13, 20, 21, 22

  Hint for #20ab: $\dfrac{z^{n+1}-1}{z-1} = \dfrac{z^{n+1}-1}{z-1} \cdot \dfrac{z^{-1/2}}{z^{-1/2}} = \dfrac{z^{n+1/2}-z^{-1/2}}{z^{1/2}-z^{-1/2}}.$

  Hint for #20b: Use the following sum-to-product formula once:
  $$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right).$$
  Hint for #21: If $|z| = 1$ then
  $$\left|\frac{1-z^n}{1-z}\right| = \left|\frac{z^{-n/2}-z^{n/2}}{z^{-1/2}-z^{1/2}} \cdot \frac{z^{n/2}}{z^{1/2}}\right| = \left|\frac{z^{-n/2}-z^{n/2}}{z^{-1/2}-z^{1/2}}\right|.$$
- (1.6) 2, 3, 4, 5, 6 (no justification is needed for these problems)
- (Not complex analysis yet, but will be useful later) The gamma function is defined by the integral
  $$\Gamma(s) = \int_0^\infty t^{s-1}e^{-t}\,dt.$$
  (for now, $s \in \mathbb{R}$, but later we will take $s \in \mathbb{C}$) As you can check by the comparison test, this integral is absolutely convergent for $s > 0$.
  (a) Compute $\Gamma(1)$.
  (b) Prove that $s\Gamma(s) = \Gamma(s+1)$. *Hint*: integration by parts.
  (c) Prove that $\Gamma(n+1) = n!$ for $n \in \mathbb{N}$.
  (d) Compute $\Gamma(\frac{1}{2})$ using the Gaussian integral $\int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi}$.