

MATH 131A HOMEWORK 7

General hint: every question uses the intermediate value theorem.

- (1) Suppose that f and g are continuous on $[a, b]$ such that $f(a) \geq g(a)$ and $f(b) \leq g(b)$. Prove that there exists a point $c \in [a, b]$ such that $f(c) = g(c)$.
- (2) Suppose that f is a real-valued continuous function on $\mathbb{R}$ and that $f(a)f(b) < 0$ for some $a, b \in \mathbb{R}$. Prove that there exists a point c between a and b such that $f(c) = 0$.
- (3) Suppose that f is continuous on $[0, 2]$ and that $f(0) = f(2)$. Prove that there exists a point $c \in [0, 2]$ such that $f(c) = f(c + 1)$.
Hint: Consider the function $g(x) = f(x + 1) - f(x)$ on the interval $[0, 1]$.