MATH 131A HOMEWORK 5

(1) Let (a_n) and (b_n) be sequences and suppose that there exists $N \in \mathbb{N}$ such that $a_n \le b_n$ for all $n \ge N$. Prove that $\limsup a_n \le \limsup b_n$.

(2) Let (a_n) and (b_n) be sequences.
   (a) Show that
   $$\sup\{a_n + b_n : n > N\} \le \sup\{a_n : n > N\} + \sup\{b_n : n > N\}.$$
   Note: this is not the same thing you proved on your last homework assignment.
   (b) Show that $\limsup(a_n + b_n) \le \limsup a_n + \limsup b_n$.

(3) Prove that (a_n) is bounded if and only if $\limsup |a_n|$ is finite.

(4) Using only the comparison test and facts about p-series, determine which of the following series is convergent (and justify your answers). Prove any inequalities you use which are not immediately obvious (induction may be helpful).
   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\log n}$
   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$

(5) Prove that if $\sum |a_n|$ converges and (b_n) is a bounded sequence then $\sum a_n b_n$ converges. *Hint:* use the Cauchy criterion.

Extra Practice (do not turn these in)

(1) Exercises 12.2, 12.3, 12.5 on p. 82 - fun with lim inf and lim sup

(2) In exercises 14.1–14.4 on p. 104, think about how you would prove that each series converges or diverges. For which series does the comparison test apply easily? Repeat (4) for some of these series.

(3) Exercises 14.5, 14.7, and 14.8 on p. 104.