

Homework 35, due December 6

(1) (Page 466, problem 3)

(a) Suppose  $j/r$  and  $j_1/r_1$  are two distinct rational numbers, with  $0 < r < n$  and  $0 < r_1 < n$  ( $j, j_1, r, r_1$  are integers). Show that

$$\left| \frac{j_1}{r_1} - \frac{j}{r} \right| > \frac{1}{n^2}.$$

(b) Suppose, as in Shor's algorithm, that we have

$$\left| \frac{c}{2^m} - \frac{j}{r} \right| < \frac{1}{2n^2} \quad \text{and} \quad \left| \frac{c}{2^m} - \frac{j_1}{r_1} \right| < \frac{1}{2n^2}.$$

Show that  $j/r = j_1/r_1$ .

(2) Let  $f(x) = x^4 - x - 1$ . Use the extended Euclidean algorithm for polynomials to find  $g(x), h(x)$  such that

$$1 = f(x)g(x) + (x^7 - 1)h(x).$$

(3) Read the Wikipedia page for [post-quantum cryptography](#). Click at least two links on the page that interest you, and read those articles as well. Write five things you learned or found interesting.