Homework 34, due December 4

(1) Compute the discrete Fourier transform $F(x)$ of the sequence

$$1, 2, 3, 2, 1, 2, 3, 2$$

for integers x with $0 \leq x < 8$. If it is helpful, recall that

$$e^{i\theta} = \cos(\theta) + i\sin(\theta)$$

for $\theta \in \mathbb{R}$.

(2) (Page 466, problem 2a) Let $0 < s \leq m$. Fix an integer c_0 with $0 \leq c_0 < 2^s$. Show that

$$\sum_{\substack{0 \leq c < 2^m \\ c \equiv c_0 \pmod{2^s}}} e^{\frac{2\pi i c x}{2^m}} = 0$$

if $x \not\equiv 0 \pmod{2^{m-s}}$ and that this sum equals

$$2^{m-s} e^{2\pi i x c_0 / 2^m}$$

if $x \equiv 0 \pmod{2^{m-s}}$. (Hint: Write $c = c_0 + j2^s$. You might also need the formula for a geometric series.)

(3) Read Scott Aarsonson's article The Limits of Quantum Computers and write five things that you learned or found interesting.