

Homework 3, due September 11

- (1) Compute $\gcd(455, 1235)$ and $\gcd(2415, 6909)$ by hand. Find integers x, y, z, w such that $455x + 1235y = 65$ and $2415z + 6909w = 63$.
- (2) The Fibonacci numbers are given by $F_1 = 1, F_2 = 1, F_n = F_{n-1} + F_{n-2}$. True or false: Any two consecutive Fibonacci numbers are relatively prime. If true, prove it. If false, give a counterexample.
- (3) Let a and b be integers. Assume n is a prime. Using the definition of congruence modulo n , prove that $a \equiv b \pmod{n}$ if and only if $a^2 + b^2 \equiv 2ab \pmod{n}$. (It is also true that $a \equiv b \pmod{n}$ if and only if $a^2 + b^2 \equiv 2ab \pmod{n^2}$, without any assumption that n is prime. You should check this, but you don't have to turn this in for homework.)